Summer Term 2015
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Approximation Algorithms

Exercise Sheet 1

Exercise 1.1:
Prove the NP-completeness of the following problems:

(i) INSTANCE: An instance of 4SAT.
QUESTION: Is there a truth assignment making at least one literal true and at least one literal false in each clause?

(ii) INSTANCE: An instance of 3SAT.
QUESTION: Is there a truth assignment making at least one literal true and at least one literal false in each clause?

(iii) INSTANCE: An undirected graph $G = (V, E)$ and an integer k.
QUESTION: Is there an $X \subseteq V$ with $|X| \leq k$ such that $|\delta(X)| \geq k$?

Hint: Use (i) to prove (ii) and (ii) to prove (iii).

(2+2+4 points)

Exercise 1.2:
Show that the following problem is NP-complete: Let integers m and n, a subset $B \subseteq \{1, \ldots, m\} \times \{1, \ldots, n\}$ and a finite dictionary $D \subseteq \Sigma^*$ on some alphabet Σ be given. Set $W := \{1, \ldots, m\} \times \{1, \ldots, n\} \setminus B$. Decide if there is a mapping $\varphi : W \longrightarrow \Sigma$ such that all maximal words $(\varphi(i, j), \ldots, \varphi(i, j + k))$ and $(\varphi(i, j), \ldots, \varphi(i + k, j))$ are in the dictionary D.

(4 points)

Exercise 1.3:
Formulate linear-time $\frac{1}{2}$-factor approximation algorithms for the following optimization problems and prove performance ratio as well as running time:

(i) Given a directed graph G with non-negative edge weights, find an acyclic subgraph of maximum weight.

(ii) Given an undirected, unweighted graph G, determine $v, w \in V(G)$ such that their distance is maximum.

(2+2 points)

Please turn in your solutions on Tuesday, **April 21st**, before the lecture.