Approximation Algorithms

Exercise Sheet 1

Exercise 1.1:
Prove the $NP$-completeness of the following problems:

(i) **INSTANCE:** An instance of 4Sat.
**QUESTION:** Is there a truth assignment making at least one literal true and at least one literal false in each clause?

(ii) **INSTANCE:** An instance of 3Sat.
**QUESTION:** Is there a truth assignment making at least one literal true and at least one literal false in each clause?

(iii) **INSTANCE:** An undirected graph $G = (V,E)$ and an integer $k$.
**QUESTION:** Is there an $X \subseteq V$ with $|X| \leq k$ such that $|\delta(X)| \geq k$?

*Hint:* Use (i) to prove (ii) and (ii) to prove (iii).

(2+2+4 points)

Exercise 1.2:
Show that the following problem is $NP$-complete: Let integers $m$ and $n$, a subset $B \subseteq \{1,\ldots,m\} \times \{1,\ldots,n\}$ and a finite dictionary $D \subseteq \Sigma^*$ on some alphabet $\Sigma$ be given. Set $W := \{1,\ldots,m\} \times \{1,\ldots,n\} \setminus B$. Decide if there is a mapping $\varphi : W \rightarrow \Sigma$ such that all maximal words $(\varphi(i,j),\ldots,\varphi(i,j+k))$ and $(\varphi(i,j),\ldots,\varphi(i+k,j))$ are in the dictionary $D$.

(4 points)

Exercise 1.3:
Formulate linear-time $\frac{1}{2}$-factor approximation algorithms for the following optimization problems and prove performance ratio as well as running time:

(i) Given a directed graph $G$ with non-negative edge weights, find an acyclic subgraph of maximum weight.

(ii) Given an undirected, unweighted graph $G$, determine $v,w \in V(G)$ such that their distance is maximum.

(2+2 points)

Please turn in your solutions on Tuesday, **April 21st**, before the lecture.