Exercise Set 5

Exercise 5.1:
Given a connected undirected graph $G = (V, E)$, a set $T \subseteq V$ with $|T| = 3$ and a cost function $c : E \to \mathbb{R}_{\geq 0}$, show how to compute a shortest Steiner tree for $T$ in $G$ in $O(|V| \log |V| + |E|)$ time.

(4 points)

Exercise 5.2:
Let $T \subseteq \mathbb{R}^2$ be a finite set of terminals located on $k$ parallel horizontal lines (i.e. $|\{y(t) : t \in T\}| = k$). We assume that the elements of $T$ are sorted by their $x$-coordinate in non-decreasing order. Prove:

(a) If $k = 2$, a shortest rectilinear Steiner tree for $T$ can be found in linear time.

(b) If $k$ is constant and on each of the $k$ parallel lines there is a terminal with $x$-coordinate $\min \{x(t) : t \in T\}$, a shortest rectilinear Steiner tree for $T$ can be found in linear time.

(4 + 2 points)

Exercise 5.3:
Let $Y$ be a Steiner tree for terminal set $T$ in which all leaves are terminals. Prove that $\sum_{t \in T} (|\delta_Y(t)| - 1) = k - 1$, where $k$ is the number of full components of $Y$.

(2 points)

Exercise 5.4:
Given a finite set $T \subseteq \mathbb{R}^2$, show how

a) $\text{CLIQUE}(T) := \frac{1}{|T| - 1} \sum_{(x,y), (x',y') \in T} (|x - x'| + |y - y'|)$

b) $\text{STAR}(T) := \min_{(x',y') \in \mathbb{R}^2} \sum_{(x,y) \in T} (|x - x'| + |y - y'|)$

can be computed in $O(|T| \log |T|)$ time.

(2+2 points)

Deadline: Thursday, May 15, before the lecture.
The websites for lecture and exercises are linked at

http://www.or.uni-bonn.de/lectures/ss14/ss14.html
In case of any questions feel free to contact me at scheifele@or.uni-bonn.de.