Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2014

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Exercise Sheet 10

Exercise 10.1:
Let $V \subset \mathbb{R}^2$ be an instance of the EUCLIDEAN TSP and let T be a tour for V. For any line segment l of length s not containing any point of V, there is a tour for V whose length exceeds the length of T by at most $3s$ and which crosses l at most twice.

(4 points)

Exercise 10.2:
Describe a polytope P with a polynomial number of variables and constraints such that the projection of P to some of the variables is the subtour polytope.

(4 points)

Exercise 10.3:
Consider the following algorithm for the ASYMMETRIC TSP with triangle inequality: Given a complete graph G on n vertices and $c : E(G) \to \mathbb{R}_+$, find a directed cycle C in G minimizing $\frac{c(E(C))}{|C|}$ and add the edges of C to the solution. Remove all but one of the vertices of C from G and proceed recursively until G is reduced to a single vertex. Prove that this is a $(2 \ln n)$-approximation algorithm for the ATSP.

Hint: You may use that the cycle C can be found in polynomial time.

(4 points)

Exercise 10.4:
Consider the ASYMMETRIC TSP with triangle inequality as in Exercise 10.3.

(i) Show that a subgraph H of G with $|\delta_H^+(v)| = |\delta_H^-(v)| = 1$ for all $v \in V(G)$ and minimum cost $c(E(H))$ can be found in polynomial time.

(ii) Use this to give a $(\log_2 n)$-approximation algorithm for the ATSP.

(4 points)

Please hand in your solutions before the lecture on Tuesday, **July 1st, at 2:15 PM**.