Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2014

Prof. Dr. J. Vygen
S. Spirkl

Exercise Sheet 9

Exercise 9.1:
Consider the Euclidean Traveling Salesman Problem.

(i) Prove that in an optimal solution of the problem no two edges cross each other.

(ii) Find a class of instances for which the quality of the Double-Tree Algorithm is arbitrarily close to the guaranteed factor of 2.

(2 + 2 points)

Exercise 9.2:
Consider the following two formulations of the Asymmetric TSP:

(a) Given a strongly connected digraph G on n vertices with edge weights $c : E(G) \to \mathbb{R}_+$, find a minimum-cost connected multi-subgraph H such that $0 < |\delta^+_H(v)| = |\delta^-_H(v)|$ for all $v \in V(G)$, i. e. H is an Eulerian spanning multi-subgraph of G. A *multi-subgraph* arises from a subgraph by possibly duplicating edges.

(b) Given a complete digraph G on n vertices with edge weights $c : E(G) \to \mathbb{R}_+$ satisfying the triangle inequality , i. e. $c((x, y)) + c((y, z)) \geq c((x, z))$ for all distinct $x, y, z \in V(G)$, find a minimum cost directed Hamiltonian circuit.

Prove the following:

(i) Formulations (a) and (b) are equivalent, i. e. there are L-reductions in both directions with $\alpha = \beta = 1$.

(ii) For every $n \geq 3$, there is an instance of (a) in which every optimum solution uses one of the edges at least $n - 2$ times.

(2 + 2 points)

Exercise 9.3:
Show that the Euclidean Steiner Tree Problem has an approximation scheme.
Hint: Modify Arora's algorithm.

(4 points)

Exercise 9.4:
Given a complete graph G, $s, t \in V(G)$ and weights $c : E(G) \to \mathbb{R}_+$ satisfying the triangle inequality, consider the problem to find an s-t-path containing all vertices of G of minimum total weight.

(i) Propose a 2-approximation algorithm for this problem.

(ii) Generalize Christofides' Algorithm to obtain a $\frac{5}{3}$-approximation algorithm.

(1 + 3 points)

Please hand in your solutions before the lecture on Tuesday, **June 17th, at 2:15 PM**.