Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2014

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Exercise Sheet 8

Exercise 8.1:
The Metric Bipartite Traveling Salesman Problem is the problem of finding a Hamiltonian circuit of minimum cost in a bipartite graph G with a nonnegative cost function c satisfying $c(\{a,b\})+c(\{a',b\})+c(\{a',b'\}) \geq c(\{a,b'\})$ for $\{a,b\}, \{a',b\}, \{a,b'\}, \{a',b'\} \in E(G)$.
Prove that for any k, if there is a k-factor approximation algorithm for the Metric Bipartite Traveling Salesman Problem, there is also a k-factor approximation algorithm for the Metric Traveling Salesman Problem.

Hint: Given an instance (G,c) of the Metric TSP, construct an instance (H,d) of the Metric Bipartite TSP where $V(H) := V(G) \times \{1,2\}$ and $d\big(\{(v,1),(w,2)\}\big) \in \big\{c(\{v,w\}), 0\big\}$.

(4 points)

Exercise 8.2:
Consider the following algorithm for the Symmetric Traveling Salesman Problem with triangle inequality:
Start with an arbitrary city $u \in V$. Find a shortest edge $e = \{u,v\} \in \binom{V}{2}$ connecting u to another city v. This yields a subtour $T = (u,v,u)$. Let $U := V \setminus \{u,v\}$. Repeat the following steps until $U = \emptyset$:

(i) Find $w \in U$ with shortest distance to one of the nodes in T.
(ii) Add w to T between two neighbouring nodes $i,j \in T$ (by deleting edge $\{i,j\}$ and connecting i and j with w), such that the cost of the new tour is minimized, i.e. find neighbouring $i,j \in T$ such that that $d(i,w) + d(w,j) - d(i,j)$ is minimum. Remove w from U afterwards.

Show that this is a 2-approximation algorithm.

(4 points)

Exercise 8.3:
Let G be a complete undirected graph in which all edge lengths are either 1 or 2. Give a $\frac{4}{3}$-approximation algorithm for the TSP in this special case.

Hint: Start by finding a minimum weight simple perfect 2-matching in G, i. e. a subset M of edges such that each vertex is incident to exactly two edges in M.

(4 points)

Please hand in your solutions before the lecture on Tuesday, **June 3rd, at 2:15 PM**.