Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2014

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Exercise Sheet 7

Exercise 7.1:
Prove that the MINIMUM VERTEX COVER problem has no absolute approximation algorithm unless $P = NP$.

(3 points)

Exercise 7.2:
Prove that MAX 2-SAT is $MAXSNP$-hard.

(3 points)

Exercise 7.3:
Show that the MAXIMUM CLIQUE problem is L-reducible to the SET PACKING problem: given a set system $(U, \mathcal{S})$, find a maximum cardinality subfamily $\mathcal{R} \subseteq \mathcal{S}$ whose elements are pairwise disjoint. What does this imply for the approximability of SET PACKING?

(3 points)

Exercise 7.4:
Prove that there are constants $d \in \mathbb{N}$ and $\gamma > 0$ such that for all $n \in \mathbb{N}$, there exists a (not necessarily simple) d-regular γ-expander with $2n$ vertices.

(i) Show that $\frac{\binom{m}{k}}{\binom{n}{k}} \leq \left(\frac{m}{n}\right)^k$ for $k, m, n \in \mathbb{N}$ with $k \leq m \leq n$.

(ii) Construct a d-regular bipartite graph $G = (A \dot\cup B, E)$ with $2n$ vertices, where E is the union of d randomly chosen perfect matchings in the complete bipartite graph $(A \dot\cup B, A \times B)$. Show that there are constants $d \in \mathbb{N}$, $\gamma > 0$ and $\varepsilon > 0$ such that for all n the probability that there are sets $A' \subseteq A$, $B' \subseteq B$ with $1 \leq |A'| + |B'| \leq n$ and $|A'| \leq |B'| \leq (1+\gamma)|A'|$ satisfying $\Gamma(A') = B'$ is less than $1 - \varepsilon$.

(iii) Show that $d \in \mathbb{N}$, $\gamma > 0$ and $\varepsilon > 0$ can be chosen such that for all n, the probability that G is a γ-expander is at least ε.

Hint: You may use that $\left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{n \cdot e}{k}\right)^k$ for $k, n \in \mathbb{N}$ with $k \leq n$.

(1 + 4 + 2 points)

Please hand in your solutions before the lecture on Tuesday, **May 27th, at 2:15 PM**.