Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2014

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Exercise Sheet 6

Exercise 6.1:
Give a polynomial-time algorithm A for Bin Packing such that there is a $\delta > 0$ with $A(I) \leq \mathrm{OPT}(I) + \mathcal{O}(\mathrm{OPT}(I)^{1-\delta})$ for all instances I.

(3 points)

Exercise 6.2:
Let $PCP'(\log n, 1)$ be defined as $PCP(\log n, 1)$, except that for each decision problem in $PCP'(\log n, 1)$ there is a polynomial p such that the error probability (*soundness*) is at most $1 - \frac{1}{p(\mathrm{size}(x))}$ for input x instead of $\frac{1}{2}$. Prove that $NP \subseteq PCP'(\log n, 1)$. *Note:* You may not use the PCP-Theorem.

(3 points)

Exercise 6.3:
Prove that if Satisfiability $\in PCP(o(\log n), 1)$ (i. e. $r \in \{0,1\}^{o(\log(p(\mathrm{size(x)})))}$) then $P = NP$.

(5 points)

Exercise 6.4:
Consider the following problem: Given n variables $x_1, \ldots, x_n$ and a set of m boolean functions $\Phi = \{\Phi_1, \ldots, \Phi_m\}$ using k variables each, find a truth assignment such that the number of satisfied functions is maximized. Show that there is a constant k for which no polynomial-time 2-approximation algorithm for this problem exists (unless $P = NP$).

(5 points)

Please hand in your solutions before the lecture on Tuesday, **May 20th, at 2:15 PM**.