Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2014

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Exercise Sheet 5

Exercise 5.1:
For an instance of Bin Packing, let $(s_1, \dots, s_m)$ denote the different item sizes and $(b_1, \dots, b_m)$ their multiplicities. Denote by

$$\{T_1, \dots, T_N\} = \left\{ (k_1, \dots, k_m) \in \mathbb{Z}_+^m : \sum_{i=1}^m k_i s_i \le 1 \right\}$$

all possible configurations for a single bin, where $T_j = (t_{j1}, \dots, t_{jm})$ for $j = 1, \dots, N$. Consider the LP

$$\begin{aligned} \max \quad & yb \\ \text{s. t.} \quad & \sum_{i=1}^m t_{ji} y_i \le 1 && (j = 1, \dots, N) \\ & y_i \ge 0 && (i = 1, \dots, m) \end{aligned}$$

which is the dual of the Bin Packing LP. Show that if $s_1 > s_2 > \dots > s_m$, then this LP always has an optimum solution with $y_1 \ge y_2 \ge \dots \ge y_m$.

(3 points)

Exercise 5.2:
Show that the second step of the Fernandez-de-la-Vega-Lueker Algorithm, which partitions the instance according to the item sizes, can be implemented with running time $\mathcal{O}(n \log \frac{1}{\varepsilon})$.

Hint: You may use that the k-smallest of n numbers can be found in $\mathcal{O}(n)$ time.

(3 points)

Exercise 5.3:

(i) Prove that for any fixed $\varepsilon > 0$ there exists a polynomial-time algorithm which for any instance $I = (a_1, \dots, a_n)$ of the Bin Packing problem finds a packing using the optimum number of bins but possibly violating the capacity constraints by ε, i. e. an $f : \{1, \dots, m\} \to \{1, \dots, \mathrm{OPT}(I)\}$ with $\sum_{f(i)=j} a_i \le 1 + \varepsilon$ for all $j \in \{1, \dots, \mathrm{OPT}(I)\}$. *Hint:* Use Exercise 4.3.

(ii) Use this to show that the Multiprocessor Scheduling Problem (see Exercise 4.1) has an approximation scheme.

(4+2 points)

Please hand in your solutions before the lecture on Tuesday, **May 13th, at 2:15 PM**.