Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2014

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Exercise Sheet 4

Exercise 4.1:
Consider the Multiprocessor Scheduling Problem: Given a finite set A of tasks, a processing time $t(a) \in \mathbb{R}_+$ for each $a \in A$ and a number m of processors, find a partition $A = \dot{\bigcup}_{i=1}^{m} A_i$ of A such that $\max_{i=1}^{m} \left\{ \sum_{a \in A_i} t(a) \right\}$ is minimum.

(i) Consider a greedy algorithm that successively assigns jobs (in an arbitrary order) to the currently least used machine. Show that this is a 2-approximation algorithm.

(ii) Show that for fixed values of m the Multiprocessor Scheduling Problem has an approximation scheme.

(2 + 3 points)

Exercise 4.2:
Let $A = (a_i)_{1 \le i \le p}$ and $B = (b_j)_{1 \le j \le q}$ be two inputs of the Bin Packing problem. We write $A \subseteq B$ if there are indices $1 \le k_1 < k_2 < \cdots < k_p \le q$ with $a_i = b_{k_i}$ for $1 \le i \le p$. An algorithm for the Bin Packing problem is called monotone if for inputs A and B with $A \subseteq B$ the algorithm needs at least as many bins for B as for A. Show:

(i) Next Fit is monotone.

(ii) First Fit is not monotone.

(2 + 2 points)

Exercise 4.3:
Give an algorithm for Bin Packing restricted to instances with a constant number of different item sizes whose running time is polynomially bounded in the number n of items.

Hint: Compute which subsets of items can be packed into i bins for $i = 1, \ldots$ using dynamic programming.

(3 points)

Please hand in your solutions before the lecture on Tuesday, **May 6th, at 2:15 PM**.