Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2014

Prof. Dr. J. Vygen
S. Spirkl

Exercise Sheet 3

Exercise 3.1:
An instance of MAX-SAT is called k-satisfiable if any k of its clauses can be satisfied simultaneously. Give a polynomial-time algorithm that computes for every 2-satisfiable instance a truth assignment which satisfies at least a $\frac{\sqrt{5}-1}{2}$-fraction of the clauses.

(4 points)

Exercise 3.2:
Apply the Goemans-Williamson MAX CUT algorithm to the 5-cycle C_5 with unit weights.

(i) Show that

$$X = \begin{bmatrix} 1 & a & b & b & a \\ a & 1 & a & b & b \\ b & a & 1 & a & b \\ b & b & a & 1 & a \\ a & b & b & a & 1 \end{bmatrix}$$

with $a = -\frac{1}{4}(1+\sqrt{5})$ and $b = -\frac{1}{4}(1-\sqrt{5})$ is a feasible solution for the semidefinite programming relaxation.

(ii) Compute a possible output of the algorithm using this choice of X.

(iii) Using (i) and (ii), derive a lower bound on the integrality gap of the SDP relaxation for MAX CUT.

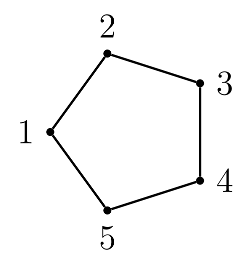

Figure: C_5

(2+1+1 points)

Exercise 3.3:
Describe exact algorithms with running time $\mathcal{O}\left(2^{n/2}\right)$ for the following problems:

1. SUBSET SUM: Given $K, n, x_1, \ldots, x_n \in \mathbb{N}$, find $S \subseteq \{1, \ldots, n\}$ with $\sum_{i \in S} x_i = K$.
2. KNAPSACK PROBLEM (where n denotes the number of items).

(4 points)

Exercise 3.4:
Consider the FRACTIONAL MULTI KNAPSACK PROBLEM: Given natural numbers

$n, m \in \mathbb{N}$ and w_i, c_{ij} as well as W_j for $1 \leq i \leq n$ and $1 \leq j \leq m$, find x_{ij} satisfying $\sum_{j=1}^{m} x_{ij} = 1$ for all $1 \leq i \leq n$ and $\sum_{i=1}^{n} x_{ij} w_i \leq W_j$ for all $1 \leq j \leq m$ such that $\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} c_{ij}$ is minimum.
State a polynomial-time combinatorial algorithm for this problem or prove NP-hardness.

(4 points)

Please hand in your solutions before the lecture on Tuesday, **April 29th, 2:15 PM**.