Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2014

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Exercise Sheet 2

Exercise 2.1:
Consider the following procedure for unweighted MINIMUM VERTEX COVER: Given a graph G, compute a DFS tree for every connected component. Return all vertices with non-zero out-degree in the tree. Show that this is a 2-approximation algorithm.

(3 points)

Exercise 2.2:
Given a set X of variables we recursively define *boolean formulas*: *true* and *false* are the boolean formulas over X of length zero and the literals are the boolean formulas of length one. The boolean formulas of length $k \geq 2$ are the strings $(\psi \wedge \psi')$ and $(\psi \vee \psi')$ where ψ and ψ' are boolean formulas of length l and l' $(l, l' \in \mathbb{N}))$ with $l + l' = k$. We extend a truth assignment $T : X \to \{\mathit{true}, \mathit{false}\}$ by setting $T(\psi \wedge \psi') := T(\psi) \wedge T(\psi')$ as well as $T(\psi \vee \psi') := T(\psi) \vee T(\psi')$. Two boolean formulas ψ and ψ' are equivalent if $T(\psi) = T(\psi')$ for every possible truth assignment T.
Given a set X of variables and a boolean formula φ over X, the LOGIC MINIMIZATION problem asks for an equivalent boolean formula of minimum length. Prove that LOGIC MINIMIZATION can be solved in polynomial time if and only if $P = NP$.

(4 points)

Exercise 2.3:
Consider the DIRECTED STEINER TREE PROBLEM: Given a edge-weighted digraph $G = (V, E)$, a set of terminals $T \subseteq V$ and a root vertex $r \in V$, find a minimum weight arborescence rooted at r that contains every vertex in T.
Show that a k-approximation algorithm for the DIRECTED STEINER TREE PROBLEM can be used to obtain a k-approximation algorithm for SET COVER.

(4 points)

Exercise 2.4:
The LP relaxation of the MINIMUM WEIGHT VERTEX COVER PROBLEM is

$$\min\{cx : M^T x \geq 1, x \geq 0\}$$

where M is the incidence matrix of an undirected graph G and $c \in \mathbb{R}_+^{V(G)}$. A *half-integral* solution for this relaxation is one with entries 0, $\frac{1}{2}$ and 1 only.

1. Show that the above LP relaxation of the MINIMUM WEIGHT VERTEX COVER PROBLEM always has a half-integral optimum solution.
2. Use this to obtain a 2-approximation algorithm. Is the analysis tight?

(4 + 1 points)

Please hand in your solutions before the lecture on Tuesday, **April 22nd, at 2:15 PM**.