Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2014

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Exercise Sheet 1

Exercise 1.1:
Prove that SATISFIABILITY remains NP-complete if each clause contains at most three literals and each variable occurs in at most three clauses.

(4 points)

Exercise 1.2:
The restriction of SATISFIABILITY to instances where each clause consists of exactly two literals is called 2-SATISFIABILITY. Prove that 2-SATISFIABILITY is in P.

(4 points)

Exercise 1.3:
Formulate linear-time 2-factor approximation algorithms for the following optimization problems and prove performance ratio as well as running time:

1. Given an undirected, unweighted graph G, determine $v, w \in V(G)$ such that their distance is maximum.
2. Given a directed graph G with non-negative edge weights, find an acyclic subgraph of maximum weight.
3. MAXIMUM-SATISFIABILITY: Given an instance for SATISFIABILITY, determine an assignment of truth values satisfying the maximum number of clauses.
4. MAXIMUM WEIGHT MATCHING: Given an undirected graph G with non-negative edge weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find a matching $M \subseteq E(G)$ such that $\sum_{e \in M} c(e)$ is maximum.

(8 points)

Please turn in your solutions before the lecture on Tuesday, **April 15th, at 2:15 PM**.