Exercise 8.1:
Show that for quadratic netlength minimization CLIQUE net models can be replaced equivalently by STAR net models by adjusting net weights.
Conclude that for quadratic netlength minimization it suffices to solve a linear equation system $Ax = b$, where the number of non-zero entries of $A$ can be bounded by a linear function in the number of pins and circuits.

(4 points)

Exercise 8.2:
Let $G = (V, E)$ be a simple undirected graph with $V = \{1, \ldots, n\}$. The Laplacian matrix $L_G$ of $G$ is the $n \times n$-matrix whose entries $l_{i,j}$, $1 \leq i, j \leq n$, are given by

$$l_{i,j} = \begin{cases} -1 & \text{if } \{i, j\} \in E, \\ |\delta(i)| & \text{if } i = j, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

(a) Prove that $L_G$ is positive semidefinite, that is, $x^T L_G x \geq 0$ for all $x \in \mathbb{R}^n$.
(b) Let $G$ be connected and let $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$ be the eigenvalues of $L_G$. Show that $\lambda_1 = 0$ and $\lambda_2 > 0$.
(c) Show that the multiplicity of 0 as an eigenvalue of $L_G$ equals the number of connected components of $G$.

(1+1+2 points)

Exercise 8.3:
Let $G$ be a graph as in Exercise 8.2 and let $\lambda_2$ be the second smallest eigenvalue of $L_G$. For $\emptyset \neq A \subset V(G)$ define the sparsity of $A$ as

$$sp(A) := \frac{|\delta(A)|}{\min\{|A|, |V(G)\setminus A|\}}.$$ 

Show that $\lambda_2 \leq 2 \cdot \min\{sp(A) : \emptyset \neq A \subset V(G)\}$.

(4 points)
Exercise 8.4:
Show that the SINGLE ROW ALGORITHM can be used to minimize the (linear) bounding box netlength instead of the quadratic movement for the SINGLE ROW PLACEMENT WITH FIXED ORDERING.

(4 points)

Deadline: Thursday, June 27th, before the lecture.