Exercise Sheet 11

Exercise 11.1:
Show that the problem of finding a minimum-cost \(s-t\)-path in an undirected graph \(G\) with edge costs \(c : E(G) \rightarrow \mathbb{R}_{\geq 0}\) can be regarded as a special case of the \textsc{Survivable Network Design Problem}. Does \textsc{Jain’s Algorithm} solve it optimally?

(3 points)

Exercise 11.2:
Show that in a slight variant of \textsc{Jain’s Algorithm} the number of iterations in which we have to solve an LP can be bounded by

(a) \(2n^2\)

(b) \(2n\)

where \(n := |V(G)|\). For this we set \(x_e := x_e + \lfloor y_e \rfloor\) for all \(e\) if some \(y_e \geq 1\), otherwise we update \(x\) as before.

Hint:

(a) Conclude from Lemma 20.32 that in the second case all but \(2n - 2\) edges can be deleted.

(b) Delete one more edge in each iteration.

(3 + 2 points)

Exercise 11.3:
Let \(PCP'(\log n, 1)\) be defined as \(PCP(\log n, 1)\) but with error probability \(\text{soundness} 1 - \frac{1}{\text{size}(x)}\) for input \(x\) instead of \(\frac{1}{2}\). Prove that \(NP \subseteq PCP'(\log n, 1)\).

Note: You may not use the PCP-Theorem.

(4 points)

Please return your solutions before the lecture on Tuesday, \textbf{July 2nd, 2:15 PM}. 