Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2013

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Exercise Sheet 10

Exercise 10.1:
Let $G = (V, E)$ be a graph with non-negative edge costs, and let $S \subset V$ and $R \subset V$ be disjoint vertex sets ("senders" and "receivers"). Consider the problem of finding a minimum cost subgraph of G that contains a path connecting each receiver to a sender.

(i) Show that this problem is NP-hard.

(ii) Prove: The special case where $S \cup R = V$ is in P.

(iii) Give a 2-approximation algorithm for the general case.

(2 + 2 + 2 points)

Exercise 10.2:
Find an optimum basic solution x for the Survivable Network Design LP

$$\min\left\{\sum_{e\in E(G)} c(e)x_e : x\big(\delta(S)\big) \geq 1 \ \big(\forall \varnothing \neq S \subsetneq V(G)\big),\ 0 \leq x_e \leq 1 \ \big(\forall e \in E(G)\big)\right\}$$

where G is the *Petersen graph* (cf. Figure 1). Find a maximal laminar family $\mathcal{B}$ of tight sets with respect to x s.t. the vectors χ_B ($B \in \mathcal{B}$) are linearly independent (cf. Lemma 20.32).

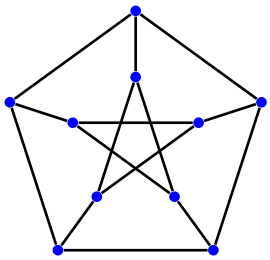

Figure 1: *The Petersen graph*

(4 points)

Exercise 10.3:
Prove: The integrality ratio of the Survivable Network Design LP is at least 2.

Note: Jain's Algorithm then implies that the integrality ratio is equal to 2.

(2 points)

Please return your solutions before the lecture on Tuesday, **June 25th, 2:15 PM**.