Exercise Sheet 10

Exercise 10.1:
Let \( G = (V, E) \) be a graph with non-negative edge costs, and let \( S \subseteq V \) and \( R \subseteq V \) be disjoint vertex sets ("senders" and "receivers"). Consider the problem of finding a minimum cost subgraph of \( G \) that contains a path connecting each receiver to a sender.

(i) Show that this problem is \( \text{NP} \)-hard.

(ii) Prove: The special case where \( S \cup R = V \) is in \( P \).

(iii) Give a 2-approximation algorithm for the general case.

(2 + 2 + 2 points)

Exercise 10.2:
Find an optimum basic solution \( x \) for the Survivable Network Design LP

\[
\min \left\{ \sum_{e \in E(G)} c(e)x_e : x(\delta(S)) \geq 1 \left( \forall \emptyset \neq S \subseteq V(G) \right), 0 \leq x_e \leq 1 \left( \forall e \in E(G) \right) \right\}
\]

where \( G \) is the Petersen graph (cf. Figure 1). Find a maximal laminar family \( B \) of tight sets with respect to \( x \) s.t. the vectors \( \chi_B \) (\( B \in B \)) are linearly independent (cf. Lemma 20.32).

![Figure 1: The Petersen graph](image)

(4 points)

Exercise 10.3:
Prove: The integrality ratio of the Survivable Network Design LP is at least 2.

Note: Jain’s Algorithm then implies that the integrality ratio is equal to 2.

(2 points)

Please return your solutions before the lecture on Tuesday, June 25th, 2:15 PM.