Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2013

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Exercise Sheet 8

Exercise 8.1:
Describe a polytope P with a polynomial number of variables and constraints (a so-called *extended formulation*) such that the projection of P to some of the variables is the subtour polytope.

(4 points)

Exercise 8.2:
Let $G = (V, E)$ be a 2-vertex-connected graph and (V, S) a DFS-tree in G rooted at $r \in V$. For each edge $e = \{v, w\} \in E \setminus S$, let w.l.o.g. v be on the r-w-path in (V, S), and let v' be the successor of v on this path. Add e to R; moreover if $|\delta(v)| \geq 3$ and $e' = \{v, v'\}$ has not yet been added to R, then add also e' to R and $\{e, e'\}$ to $\mathcal{P}$ (cf. Figure 1). Prove that the resulting $(R, \mathcal{P})$ is a *removable pairing* in G.

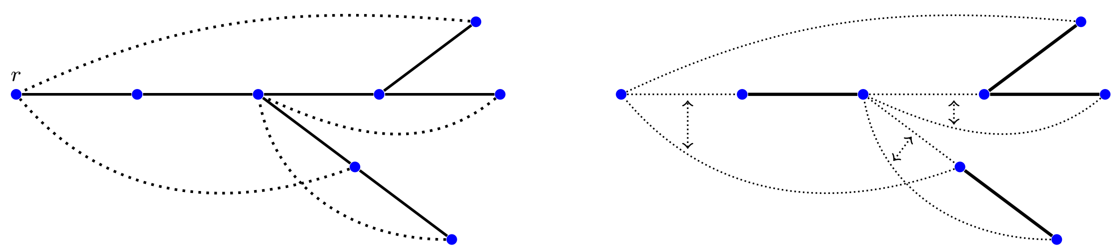

Figure 1: *A 2-connected graph with a DFS tree (left: solid edges) and a removable pairing (right: dotted edges are in R; arrows indicate pairs).*

(4 points)

Exercise 8.3:
Consider the Traveling Salesman Problem on subcubic graphs (i.e. graphs with maximum degree 3). Propose a $\frac{4}{3}$-approximation for this problem.

Hint: Use Exercise 8.2 and the Lemma of Mömke and Svensson.

(4 points)

Exercise 8.4:
Let G be a complete undirected graph in which all edge lengths are either 1 or 2. Give a $\frac{4}{3}$-approximation algorithm for the TRAVELING SALESMAN PROBLEM in this special case.

Hint: Find a minimum 2-matching in G, i.e. a subset M of edges such that every vertex has exactly 2 edges of M incident at it, and consider the cycles in M.

(4 points)

Please return your solutions before the lecture on Tuesday, **June 11th, 2:15 PM**.