Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2013

Prof. Dr. J. Vygen
P. Ochsendorf, M. Sc.

Exercise Sheet 7

Exercise 7.1:
Consider the following algorithm for the SYMMETRIC TRAVELING SALESMAN PROBLEM with triangle inequality:
Start with an arbitrary city $u \in V$. Find a shortest edge $e = \{u, v\} \in \binom{V}{2}$ connecting u to another city v. This yields a subtour $T = (u, v, u)$. Let $U := V \setminus \{u, v\}$. Repeat the following steps until $U = \emptyset$:

(i) Find $w \in U$ with shortest distance to one of the nodes in T.

(ii) Add w to T between two neighbouring nodes $i, j \in T$ (by deleting edge $\{i, j\}$ and connecting i and j with w), such that the cost of the new tour is minimized, i.e. find neighbouring $i, j \in T$ such that that $d(i, w) + d(w, j) - d(i, j)$ is minimum. Remove w from U afterwards.

Show that this is a 2-approximation algorithm.

(4 points)

Exercise 7.2:
Consider the EUCLIDEAN TRAVELING SALESMAN PROBLEM.

(i) Prove that in an optimal solution of the problem no two edges cross each other.

(ii) Find a class of instances for which the quality of the DOUBLE-TREE ALGORITHM is arbitrarily close to the guaranteed factor of 2.

(2 + 3 points)

Exercise 7.3:
Given a complete graph G, $s, t \in V(G)$ and weights $c : E(G) \to \mathbb{R}_+$ satisfying the triangle inequality, consider the problem to find an s-t-path containing all vertices of G of minimum total weight.

(i) Propose a 2-approximation for this problem.

(ii) Generalize the CHRISTOFIDES' ALGORITHM to obtain a $\frac{5}{3}$-approximation.

(3 + 4 points)

Please return your solutions before the lecture on Tuesday, **June 4th, 2:15 PM**.