Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2013

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Exercise Sheet 6

Exercise 6.1:
Describe a polynomial-time algorithm which optimally solves any instance of the TRAVELING SALESMAN PROBLEM that is the metric closure of a weighted tree.

(4 points)

Exercise 6.2:
The METRIC BIPARTITE TRAVELING SALESMAN PROBLEM is the problem of finding a Hamiltonian circuit of minimum cost in a bipartite graph G with a nonnegative cost function c satisfying $c(\{a,b\}) + c(\{a',b\}) + c(\{a',b'\}) \geq c(\{a,b'\})$ for $\{a,b\}, \{a',b\}, \{a,b'\}, \{a',b'\} \in E(G)$.
Prove that for any k, if there is a k-factor approximation algorithm for the METRIC BIPARTITE TRAVELING SALESMAN PROBLEM, there is also a k-factor approximation algorithm for the METRIC TRAVELING SALESMAN PROBLEM.

Hint: Given an instance (G,c) of the METRIC TSP, construct an instance (H,d) of the METRIC BIPARTITE TSP where $V(H) := V(G) \times \{1,2\}$ and $d\big(\{(v,1),(w,2)\}\big) \in \big\{c(\{v,w\},0)\big\}$.

(4 points)

Exercise 6.3:
Find a class of instances of the METRIC TRAVELING SALESMAN PROBLEM for which CHRISTOFIDES' ALGORITHM returns a tour whose length is arbitrarily close to $\frac{3}{2}$OPT.

(4 points)

Please return your solutions before the lecture on Tuesday, **May 28th, 2:15 PM**.