Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2013

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Exercise Sheet 5

Exercise 5.1:
Let $A = (a_i)_{1 \le i \le p}$ and $B = (b_j)_{1 \le j \le q}$ be two inputs of the Bin Packing Problem. We write $A \subseteq B$ iff there are indices $1 \le k_1 < k_2 < \cdots < k_p \le q$ with $a_i = b_{k_i}$ for $1 \le i \le p$. An algorithm for the Bin Packing problem is called monotone if for inputs A and B with $A \subseteq B$ the algorithm needs at least as many bins for B as for A. Show:

(a) Next Fit is monotone.

(b) First Fit is not monotone.

(4 points)

Exercise 5.2:
Show that Bin Packing with a fixed number of different item sizes can be solved in polynomial time.

Hint: Compute which subsets of items can be packed into i bins for $i = 1, \ldots$ using dynamic programming.

(4 points)

Exercise 5.3:
Consider the following Multiprocessor Scheduling Problem: Given a finite set A of tasks, a number $t(a) \in \mathbb{R}_+$ for each $a \in A$ (the *processing time*) and a number m of processors, find a partition $A = \dot\bigcup_{i=1}^m A_i$ of A into m pairwise disjoint sets A_i such that $\max_{i=1}^m \left\{ \sum_{a \in A_i} t(a) \right\}$ is minimum.

(i) Consider a greedy algorithm that successively assigns jobs (in an arbitrary order) to the currently least used machine. Show that such an algorithm is a 2-approximation algorithm.

(ii) Show that for fixed values of m the Multiprocessor Scheduling Problem has an approximation scheme.

(4 points)

Please return your solutions before the lecture on Tuesday, **May 14th, 2:15 PM**.