Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2013

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Exercise Sheet 4

Exercise 4.1:
Let $m \in \mathbb{N}$ be a constant. Consider the following scheduling problem: Given $n \in \mathbb{N}$ (the *number of jobs*), costs $c : \{1, \dots, n\} \times \{1, \dots, m\} \in \mathbb{N} \cup \{0\}$, and capacities $T_j \in \mathbb{N} \cup \{0\}$ for $1 \leq j \leq m$, find an assignment $f : \{1, \dots, n\} \to \{1, \dots, m\}$ with

$$\left|\left\{i \in \{1, \dots, n\} : f(i) = j\right\}\right| \leq T_j \qquad \text{for } 1 \leq j \leq m$$

and minimum total cost $\sum_{i=1}^{n} c\big(i, f(i)\big)$.
Show that this problem has a fully polynomial approximation scheme.

(6 points)

Exercise 4.2:
Consider the following generalization of the Knapsack Problem: Given $n \in \mathbb{N}$ items with weight $w_j \in \mathbb{N}$ for $1 \leq j \leq n$ together with $W \in \mathbb{N}$ and profits $p_{ij} \in \mathbb{N}$ for $i, j \in \mathbb{N}$, $1 \leq i < j \leq n$, determine a subset $S \subseteq \{1, \dots, n\}$ for which $\sum_{i \in S} w_i \leq W$ and the overall profit $\sum_{i,j \in S:\, i<j} p_{ij}$ is maximum.

1. Show that this variant of the Knapsack Problem is strongly NP-hard.
2. Deduce that no fully polynomial approximation scheme for this problem exists unless $P = NP$.

(3 + 2 points)

Exercise 4.3:
Suppose that in an instance $a_1, \dots, a_n$ of the Bin Packing problem we additionaly require $a_i > \frac{1}{3}$ for $1 \leq i \leq n$.

1. Reduce the problem to the Cardinality Matching problem.
2. Describe an algorithm that solves the problem in $O(n \log(n))$.

(2 + 3 points)

Please return your solutions before the lecture on Tuesday, **May 7th, 2:15 PM**.