Research Institute for Discrete Mathematics
Approximation Algorithms
Summer Term 2013

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Exercise Sheet 2

Exercise 2.1:
Given an undirected graph G and $k \in \mathbb{N}$, the Stable Set problem asks whether a set $S \subseteq V(G)$ exists such that $|S| \geq k$ and $E\big(G[S]\big) = \emptyset$. Prove that Stable Set is NP-complete even if restricted to graphs whose maximum degree is 4.

Hint: Use Exercise 1.1.

(4 points)

Exercise 2.2:
Given a set X of variables we recursively define *Boolean Formulas*: *true* and *false* are the Boolean formulas over X of length zero and the literals are the Boolean formulas of length one. The Boolean formulas of length $k \geq 2$ are the strings $(\psi \wedge \psi')$ and $(\psi \vee \psi')$ where ψ and ψ' are Boolean formulas of length l and l' $(l, l' \in \mathbb{N})$) with $l + l' = k$. We extend a truth assignment $T : X \to \{\mathit{true}, \mathit{false}\}$ by setting $T(\psi \wedge \psi') := T(\psi) \wedge T(\psi')$ as well as $T(\psi \vee \psi') := T(\psi) \vee T(\psi')$. Two Boolean formulas ψ and ψ' are equivalent if $T(\psi) = T(\psi')$ for every possible truth assignment T.
Given a set X of variables and a Boolean formula φ over X, the Logic Minimization problem asks for an equivalent Boolean formula of minimum length. Prove that Logic Minimization can be solved in polynomial time if and only if $P = NP$.

(4 points)

Exercise 2.3:
The LP relaxation of the Minimum Weight Vertex Cover problem is

$$\min\{cx : M^T x \geq 1, x \geq 0\}$$

where M is the incidence matrix of an undirected graph G and $c \in \mathbb{R}_+^{V(G)}$. A *half-integral* solution for this relaxation is one with entries 0, $\frac{1}{2}$ and 1 only.

1. Show that the above LP relaxation of the Minimum Weight Vertex Cover problem always has a half-integral optimum solution.

2. Use this to obtain a 2-approximation algorithm. Is the analysis tight?

(6 + 2 points)

Please return your solutions before the lecture at Tuesday, **April 23rd, at 2:15 PM**.