Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2012

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Exercise Set 13

Exercise 1:
Let C^* be an optimal solution of the TSP(1,2) problem, i.e. the traveling salesman problem where all edge lengths are 1 or 2, and let C be a solution found by the 2-OPT algorithm. Let s^* and s be the number of edges in C^* resp. C with edge length 1.

(i) Prove: $s^* \leq 2s$.

(ii) Conclude that the 2-OPT algorithm has an approximation guarantee of $\frac{3}{2}$ in this special case.

(3+2 points)

Exercise 2:
Modify Christofides' algorithm to find a traveling salesman path (a path that visits every city exactly once) whose length is at most $\frac{3}{2}$ times the length of an optimal path.

(4 points)

Exercise 3:
Consider the Cycle Cover Problem: Given an undirected graph $G = (V, E)$ and non-negative edge weights, find a set of vertex-disjoint cycles with minimum weight and the property that each $v \in V$ is contained in exactly one of the cycles. In the Directed Cycle Cover Problem, the graph is directed, and directed cycles need to be found.

(i) Find a polynomial-time algorithm for the Cycle Cover Problem.

(ii) Find a polynomial-time algorithm for the Directed Cycle Cover Problem.

Hint: A minimum weight perfect matching can be found in polynomial time.

(3+4 points)

Remark: Points achieved with these exercises are not relevant for admission to the final examination.

Please return the exercises until Tuesday, **July 10th, at 2:15 pm**.