Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2012

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Exercise Set 12

Exercise 1:
Consider the following algorithm for the Traveling Salesman Problem:
Start with an arbitrary vertex $u \in V(G)$ and subtour $T = (u)$. Find a shortest edge $e = (u, v) \in E(G)$ connecting u to another vertex, and add v to T. This yields subtour $T = (u, v, u)$. Delete v from $V(G)$, and repeat the following steps until $V(G) = \emptyset$:

1. Find $w \in V(G)$ with shortest distance to one of the nodes in T.
2. Add w to T between two neighbouring nodes $i, j \in T$ (by deleting edge (i, j) and connecting i and j with w) such that the cost of the new tour is minimized, i.e. find neighbouring $i, j \in T$ such that $d(i, w) + d(w, j) - d(i, j)$ is minimum.
   Remove w from $V(G)$.

Show that this is a 2-approximation algorithm.

(4 points)

Exercise 2:
Let c_0 be the value of an optimal solution of an instance of the Metric TSP and c_1 the cost of a second-shortest tour. Prove $\frac{c_1 - c_0}{c_0} \leq \frac{2}{n}$.

(4 points)

Exercise 3:
A weight function c on a bipartite graph G satisfies the *square inequality* if
$c(a, b) + c(a', b) + c(a', b') \geq c(a, b')$ for all $\{a, b\}, \{a', b\}, \{a, b'\}, \{a', b'\} \in E(G)$.
Consider the following problem: Given a complete bipartite graph with non-negative edge weights which satisfy the square inequality, find a Hamiltonian circuit of minimum weight.
Show that if there is an α-approximation algorithm for this problem, then there also is an α-approximation algorithm for the Metric TSP.

(*Hint: Given an instance* (G, c) *of the* Metric TSP*, construct an instance* (H, d) *of the other problem with* $V(H) := V(G) \times \{1, 2\}$ *and* $d(\{(v, 1), (w, 2)\}) = c(\{v, w\})$ *if* $\{v, w\} \in E(G)$*, and* 0 *otherwise.*)

(4 points)

Exercise 4:
Consider the EUCLIDIAN TSP, i.e. all nodes are embedded into $\mathbb{R}^2$ and the edge lengths are equal to the Euclidian distance of the vertices they connect. Prove that in an optimal solution of the problem no two edges cross each other.

(3 points)

Exercise 5:
Let G be a complete undirected graph in which all edge lengths are either 1 or 2. Give a $\frac{4}{3}$-approximation algorithm for the TRAVELING SALESMAN PROBLEM in this special class of graphs.

Hint: Find a minimum 2*-matching in* G*, i.e. a subset* M *of edges such that every vertex has exactly* 2 *edges of* M *incident at it, and consider the cycles in* M*.*

(3 points)

Please return the exercises until Tuesday, **July 3rd, at 2:15 pm**.