Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2012

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Exercise Set 11

Exercise 1:
Let $G = (V, E)$ be a graph with non-negative edge costs, and let $S \subset V$ and $R \subset V$ be disjoint vertex sets ("senders" and "receivers"). Consider the problem of finding a minimum cost subgraph of G that contains a path connecting each receiver to a sender.

(a) Prove: If $S \cup R = V$, then the problem is in P.

(b) Give a 2-factor approximation algorithm for the case $S \cup R \neq V$ (which is NP-hard).

(2+3 points)

Exercise 2:
Let $G = (V, E)$ be the grid graph on the vertex set $V = n \times n$, $n \in \mathbb{N}$, in which all edges have length 1. Let $R \subset V$ be a terminal set with $|R| = 3$.

(a) Let b be the width and h be the height of the smallest rectangle enclosing R. Show that the length of a minimum Steiner $smt(R) = h + b$.

(b) Show that in average, ranging over all possible choices of R with $|R| = 3$, $smt(R) = n$.

(1+2 points)

Exercise 3:
Consider the B-Vertex Cover Problem: Given a graph where the maximum degree of every vertex is bounded by a constant B, find a vertex cover of minimum cardinality. Show: If there exists a polynomial time approximation algorithm for the Steiner Tree Problem with performance ratio $1+\epsilon$, then there exists a polynomial time approximation algorithm for the B-Vertex Cover Problem with performance ratio $1+(B+1)\epsilon$.

(4 points)

Exercise 4:
Given an instance (K_n, c) of the Traveling Salesman Problem and an edge $e \in E(K_n)$, show that it is NP-complete to decide whether e occurs in an optimal tour.

(4 points)

Please return the exercises until Tuesday, **June 26th, at 2:15 pm**.