Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2012

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Exercise Set 10

Exercise 1:
Show that in Mehlhorns algorithm replacing the edges of the minimal spanning tree by the corresponding paths does not result in cycles.

(4 points)

Exercise 2:
Show that the contraction lemma still holds if one adds edges between terminals whose lengths are larger than 0. ("Adding" an edge means that if there already is an edge, a parallel edge is inserted.)

(3 points)

Exercise 3:
Consider the Relative Greedy algorithm.

(a) For each $k \in \mathbb{N}$, $k > 2$, find an instance of the Steiner Tree Problem for which the solution found by the algorithm is not optimal.

(b) What approximation factor does the algorithm have for $k = 5$?

(3+1 points)

Exercise 4:
The "vertex version" of the contraction lemma is wrong. To show this, define a complete graph whose edge lengths fulfill the triangle inequality, and vertex sets A, B and C such that

$$0 < mst(A) - mst(A \cup C) < mst(A \cup B) - mst(A \cup B \cup C).$$

Here $mst(X)$ for a vertex set X denotes the length of a minimum spanning tree in the graph induced by X.

(2 points)

Exercise 5:
Consider an instance $G = (V, E)$ of the Steiner Tree Problem with terminal set R and edge lengths $c : E \to \mathbb{R}_+$.
Denote the full components of an optimal k-Steiner tree $SMT_k(R)$ with $T_1^*, \ldots, T_q^*$.

(a) Suppose $V \backslash R$ forms a stable set.
Show that $mst(R) \leq 2 \cdot (smt_k(R) - loss(T_1^*, \ldots, T_q^*))$ holds in that case.

(b) Ssuppose all shortest paths between two vertices in G have length 1 or 2. Show that $mst(R) \leq 2 \cdot (smt_k(R) - loss(T_1^*, \ldots, T_q^*))$ holds in that case.

(c) Show that the Loss Contraction Algorithm achieves an approximation ratio of $1.279 r_k$ in both cases.
Hint: Use (a) and (b) in the analysis of the Loss Contraction Algorithm

(4 points)

Please return the exercises until Tuesday, **June 19th, at 2:15 pm**.