

## Exercise Set 8

**Exercise 1:**

Consider the MINIMUM MAKESPAN SCHEDULING problem with a constant number  $m$  of machines. Use the algorithm from Exercise 7.3 to obtain a PTAS for this problem.

(4 points)

**Exercise 2:**

Consider the MAXIMUM CLIQUE problem: Given a graph  $G = (V, E)$ , find a clique of maximum size in  $G$ . Show that (unless  $P = NP$ ), there exists no absolute approximation algorithm for this problem.

(4 points)

**Exercise 3:**

Show that if there is an algorithm for BIN PACKING having a guarantee of  $OPT(I) + \log^2(OPT(I))$ , then there is a fully polynomial approximation scheme for this problem.

(4 points)

**Exercise 4:**

Prove: Unless  $P = NP$ , there does not exist an absolute approximation algorithm for the STEINER TREE problem.

(4 points)

**Exercise 5:**

Describe an algorithm for the STEINER TREE problem which runs in  $O(n^3)$  for instances  $(G, c, R)$  with  $|V(G) \setminus R| \leq s$  for some constant  $s$ .

(3 points)

**Exercise 6:**

Consider the following algorithm for the STEINER TREE problem with 3 terminals  $v_1$ ,  $v_2$  and  $v_3$ : Find the shortest path  $P$  between  $v_1$  and  $v_2$  and let  $a$  be the distance of  $v_3$  to  $P$ . Then find a vertex  $z$  which minimizes  $\sum_{i=1}^3 \text{dist}(v_i, z)$  under the conditions  $\text{dist}(v_i, z) \leq \text{dist}(v_1, v_2)$  for  $i \in \{1, 2\}$  and  $\text{dist}(v_3, z) \leq a$ . The algorithm finally returns the union of the shortest paths from  $z$  to the terminals.

Show that the algorithm can be implemented in  $O(m + n \log n)$  and works correctly.

(3 points)

Please return the exercises until Tuesday, **June 5th, at 2:15 pm.**