Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2012

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Exercise Set 6

Exercise 1:
Consider the following variant of the KNAPSACK problem:
Given $n, c_1, \ldots, c_n, w_1, \ldots, w_n \in \mathbb{N}$ and a nonincreasing function $b(i) : \{1, \ldots, n\} \to \mathbb{N}$, maximize $\sum_{i=1}^{n} c_i x_i$ subject to $\sum_{i=1}^{n} w_i x_i \leq b(\sum_{i=1}^{n} x_i)$, $x_i \in \{0, 1\}$, $i = 1, \ldots, n$.
Show how this variant can be polynomially reduced to the standard KNAPSACK problem.

(4 points)

Exercise 2:
Suppose that in an instance $a_1, \ldots, a_n$ of the BIN-PACKING problem we have $a_i > \frac{1}{3}$ for each i. Reduce the problem to the CARDINALITY MATCHING problem. Then show how to solve the problem in $O(n \log n)$ time.

(3 points)

Definition:
An absolute approximation algorithm for an optimization problem is a polynomial-time algorithm A for which there exists a constant k such that $|A(I) - OPT(I)| \leq k$ holds for every instance I.

Exercise 3:
Prove that there does not exist an absolute approximation algorithm for the KNAPSACK Problem (unless $P = NP$).

(3 points)

Exercise 4:
Find an NP-hard optimization variant of PARTITION for which an absolute approximation algorithm A with $k = 1$ exists, and prove that it exists.

(4 points)

Please return the exercises until Tuesday, **May 15th, at 2:15 pm**.