Exercise Set 5

Exercise 1:
Describe exact algorithms with running times of $O(2^n)$ for the following problems:

(i) **Subset Sum**, where $n$ is the number of numbers.

(ii) **Knapsack**, where $n$ is the number of items.

(3+3 points)

Exercise 2:
Consider the **Fractional MultiKnapsack** problem: Given natural numbers $m$, $n$, $w_i$, $c_{ij}$, and $W_j$ for $1 \leq i \leq n$ and $1 \leq j \leq m$, find $x_{ij} \in [0, 1]$ satisfying $\sum_{j=1}^{m} x_{ij} = 1$ for all $1 \leq i \leq n$ and $\sum_{i=1}^{n} x_{ij} w_i \leq W_j$ for all $1 \leq j \leq m$, such that $\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} c_{ij}$ is minimum.

Provide a polynomial-time combinatorial algorithm for this problem or prove that it is $NP$-hard.

(4 points)

Exercise 3:
Prove that the greedy algorithm for the **Knapsack** Problem (take the element with the best $\frac{c_i}{w_i}$ ratio and add $i$ to $S$ until $\sum_{i \in S} w_i > W$) cannot achieve a constant approximation ratio.

(3 points)

Please return the exercises until Tuesday, **May 8th, at 2:15 pm**.