Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2012

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Exercise Set 5

Exercise 1:
Describe exact algorithms with running times of $\mathcal{O}(2^{\frac{n}{2}})$ for the following problems:

(i) Subset Sum, where n is the number of numbers.

(ii) Knapsack, where n is the number of items.

(3+3 points)

Exercise 2:
Consider the Fractional MultiKnapsack problem: Given natural numbers m, n, w_i, c_{ij}, and W_j for $1 \leq i \leq n$ and $1 \leq j \leq m$, find $x_{ij} \in [0,1]$ satisfying $\sum_{j=1}^{m} x_{ij} = 1$ for all $1 \leq i \leq n$ and $\sum_{i=1}^{n} x_{ij} w_i \leq W_j$ for all $1 \leq j \leq m$, such that $\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} c_{ij}$ is minimum.
Provide a polynomial-time combinatorial algorithm for this problem or prove that it is NP-hard.

(4 points)

Exercise 3:
Prove that the greedy algorithm for the Knapsack Problem (take the element with the best $\frac{c_i}{w_i}$ ratio and add i to S until $\sum_{i \in S} w_i > W$) cannot achieve a constant approximation ratio.

(3 points)

Please return the exercises until Tuesday, **May 8th, at 2:15 pm**.