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Exercise Set 4

Exercise 1:
Describe an algorithm which decides if a graph $G = (V, E)$ is 4-colourable with a running time of $O(|E| \cdot 2^{|V|})$.

(4 points)

Exercise 2:
Show that any 4-colourable graph with n vertices can be coloured with $O(n^{\frac{2}{3}})$ colours in polynomial time.

(4 points)

Exercise 3:
An instance of MAX-SAT is called k-satisfiable if any k of its clauses can be simultaneously satisfied. Let r_k be the infimum of the fraction of clauses that one can satisfy in any k-satisfiable instance.

(a) Prove that $r_1 = \frac{1}{2}$

(b) Prove that $r_2 = \frac{\sqrt{5}-1}{2}$
(Hint: Korte, Vygen: "Combinatorial Optimization. Theory and Algorithms.", Fourth Edition, Exercise 16.17.)

(1+4 points)

Exercise 4:
Given an undirected graph $G = (V, E)$, find a set $X \subseteq V$ maximizing $|\delta(X)|$. Consider the following algorithm: Start with $X = \emptyset$. If adding a single vertex to X or deleting a single vertex from X makes $|\delta(X)|$ larger, then do so. Repeat until no improvement is possible.

(a) Show that the algorithm runs in polynomial time.

(b) Show that this is a $\frac{1}{2}$-approximation algorithm.

(1+3 points)

Exercise 5:
Show with a reduction from 3Sat that Max-2-Sat is NP-hard.

(4 points)

Please return the exercises until Thursday, **May 3rd, at 2:15 pm**.