Exercise Set 3

Exercise 1:
Formulate fast 2-factor approximation algorithms for the following problems and prove the approximation factor:

(a) Given an undirected graph $G = (V, E)$, what is the diameter of $G$?
(The diameter of $G$ is defined as $\text{diam}(G) := \max_{v, w \in V} \text{dist}(v, w)$, where $\text{dist}(v, w)$ is the length of a shortest $v$-$w$-path.)
Hint: Linear runtime is possible.

(b) Given a directed graph $G$ with edge weights, find a directed acyclic subgraph of maximum weight.

(4+4 points)

Exercise 2:
Consider the following greedy algorithm for VERTEX COVER: Start with $C = \emptyset$. While there are still edges in $G$, choose the node in $G$ with the largest degree, add it to $C$, and delete it from $G$.

(i) Show that the algorithm never produces a solution which is more than $\log n$ times the optimum.

(ii) Find a family of graphs in which the $\log n$ bound is achieved in the limit.

(4+2 points)

Exercise 3:
Consider an optimization problem $\mathcal{P}$ and the corresponding decision problem $\mathcal{P}'$. Show that if $\mathcal{P}'$ can be solved in polynomial time, then $\mathcal{P}$ can also be solved in polynomial time.

(3 points)

Please return the exercises until Tuesday, April 24th, at 2:15 pm.