Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2012

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Exercise Set 2

Exercise 1:
Prove NP-completeness of the following problems:

(i) INSTANCE: An instance of 3SAT.
TASK: Is there a truth assignment that makes at least one literal true and at least one literal false in each clause?

(ii) INSTANCE: An undirected graph $G = (V, E)$ and an integer k.
TASK: Is there an $X \subseteq V$ with $|X| \leq k$ such that $|\delta(X)| \geq k$?

Hint: Use (i) to prove (ii).

(4+4 points)

Exercise 2:
Prove NP-completeness of the following problems:

(a) SHORTEST PATH: Given a graph $G = (V, E)$, weights $c : E \to \mathbb{Z}$, two nodes $s, t \in V$ and $k \in \mathbb{N}$. Does there exist a path from s to t with weight at most k?

(b) Given a directed graph $G = (V, E)$ and an integer k, is there an $X \subseteq V$ with $|X| \leq k$ such that every directed circuit in G contains at least one node in X?

(3+3 points)

Exercise 3:
CLIQUE is NP-complete. Is it still NP-complete (provided that $P \neq NP$) if restricted to

(a) planar graphs

(b) 2-connected graphs?

(2+2 points)

Please return the exercises until Tuesday, **April 17th, at 2:15 pm**.