Chip Design
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Exercises 12

1) Given an instance of the MIN-MAX RESOURCE SHARING PROBLEM, σ-approximate bounded block solvers are oracles $f_c : \mathbb{R}_+^{\mathcal{B}_c} \times \mathbb{R}_+$ such that for all $y \in \mathbb{R}_+^{\mathcal{R}}$ and $\mu \le 0$ we have $g_c(f_c(y,\mu)) \le \mu \mathbb{1}$ and $y^\top g_c(f_c(y,\mu)) \le \sigma \inf\{y^\top g_c(b) \,|\, b \in \mathcal{B}_c, g_c(b) \le \mu \mathbb{1}\}$ for all $c \in \mathcal{C}$ (where $\mathbb{1}$ is the all-one vector). For $\mu = \infty$ we have the usual unbounded block solvers.
Show that the RESOURCE SHARING ALGORITHM can be modified for instances with $\lambda^* \le 1$ and bounded block solvers such that it computes for $0 < \delta, \delta' < 1$ a $(\sigma(1+\delta) + \frac{\delta'}{\lambda^*})$-approximate solution in $O(\theta|\mathcal{C}|\log|\mathcal{R}|(\log|\mathcal{R}| + (\delta\delta')^{-1}\sigma))$ time (compare to Theorem 4.12). To this end show that at most $t|\mathcal{C}|$ oracle calls are sufficient.

(4 points)

2) Given a pair (G, H) of undirected graphs on the same set of vertices, capacities $u : E(G) \to \mathbb{R}_+$ and demands $b : E(H) \to \mathbb{R}_+$.
A CONCURRENT FLOW of value $\alpha > 0$ is a family $(x^f)_{f \in E(H)}$, where x^f is an s-t-flow of value $\alpha \cdot b(f)$ in $(V(G), \{(v,w),(w,v) \,|\, \{v,w\} \in E(G)\})$ for each $f = \{t,s\} \in E(H)$, and

$$\sum_{f \in E(H)} \left(x^f((v,w)) + x^f((w,v))\right) \le u(e)$$

for all $e = \{v,w\} \in E(G)$.
The MAXIMUM CONCURRENT FLOW PROBLEM is to find a concurrect flow with maximum value α.
Prove that the MAXIMUM CONCURRENT FLOW PROBLEM is a special case of the MIN-MAX RESOURCE SHARING PROBLEM. How can you implement

a) unbounded block solvers and

b) bounded block solvers?

(4 points)

3) Let $k \geq 2$ and consider the following instance of the RESOURCE SHARING PROBLEM:
$\mathcal{C} := \{c\}$, $\mathcal{R} := \{r_1, \dots, r_{k+1}\}$, $\mathcal{B}_c := \mathrm{conv}(\{b_1, b_2\})$ with

i) $g_c(b_1) := (1, 0, \dots, 0)$,

ii) $g_c(b_2) := (0, 1, \dots, 1)$,

iii) $g_c(\alpha b_1 + (1-\alpha)b_2) := \alpha g_c(b_1) + (1-\alpha)g_c(b_2)$ for $0 \leq \alpha \leq 1$.

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Assume $\sigma = 1$, i.e. the block solvers f_c, $c \in \mathcal{C}$, always returns an optimum solution. Show that computing a $(1+\omega)$-approximate solution for this instance using the RESOURCE SHARING ALGORITHM requires $\Theta(\omega^{-2} \ln k)$ phases.

(4 points)

4) Let G be a directed graph with distances $l : E(G) \to \mathbb{R}_+$, $L \in V(G)$ a vertex that is reachable from any other vertex, and $t \in V(G)$. For $v \in V(G)$ let $d(v) := \max\{0, \mathrm{dist}_l(v, L) - \mathrm{dist}_l(v, t)\}$ (where $\mathrm{dist}_l(v, w)$ denotes the length of a shortest v-w path in G with respect to l). For $e = (v, w) \in E(G)$ define $l'(e) := l(e) - d(v) + d(w)$. Let $s \in V(G)$ and P a directed s-t path in G.
Prove that $l'(e) \geq 0$ for all $e \in E(G)$ and that P is a shortest s-t path with respect to l if and only if it is a shortest s-t path with respect to l'.

(4 points)

Deadline: July 13 before the lecture (12.15 pm).

Das nächste Treffen der Mentorengruppe des Forschungsinstituts fuer Diskrete Mathematik findet am Dienstag, den 13. Juli um 18:00 Uhr im Konferenzraum des Arithmeums statt. Das Thema lautet Rechteckpackungen, alle interessierten Studenten sind herzlich eingeladen.