

Exercises 8

- 1) (a) Show that for quadratic netlength minimization (Proposition 3.10) CLIQUE net models can be replaced equivalently by STAR net models by adjusting netweights.
- (b) Conclude that for quadratic netlength minimization it suffices to solve a linear equation system $Ax = b$, where A has $O(|P| + |C|)$ non-zero entries. (4 points)

2) Let $G = (V, E)$ be a simple undirected graph with $V = \{1, \dots, n\}$. The Laplacian matrix L_G of G is the $n \times n$ -matrix whose entries $l_{i,j}, 1 \leq i, j \leq n$, are given by

$$l_{i,j} = \begin{cases} -1 & \text{if } \{i, j\} \in E, \\ |\delta(i)| & \text{if } i = j, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Prove that L_G is positive semidefinite, that is, $x^T L_G x \geq 0$ for all $x \in \mathbb{R}^n$.
- (b) Let G be connected and let $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$ be the eigenvalues of L_G . Show that $\lambda_1 = 0$ and $\lambda_2 > 0$.
- (c) Show that the multiplicity of 0 as an eigenvalue of L_G equals the number of connected components of G .

(4 points)

3) Let G be a graph as in 2) and let λ_2 be the second smallest eigenvalue of L_G . For $\emptyset \neq A \subsetneq V(G)$ define the sparsity of A as

$$sp(A) := \frac{|\delta(A)|}{\min\{|A|, |V(G) \setminus A|\}}.$$

Show that $\lambda_2 \leq 2 \min_{\emptyset \neq A \subsetneq V(G)} sp(A)$.

(4 points)

4) Consider the fractional MULTISECTION PROBLEM for $m = 2$ regions. Show that there is a simple $O(n \log n)$ -time algorithm, not using network flows, which computes an optimum fractional partition in which all but one circuits are assigned to only one region.

(4 points)

Deadline: June 15 before the lecture (12.15 pm).

Das nächste Treffen der Mentorengruppe des Forschungsinstituts für Diskrete Mathematik findet am 15. Juni um 18:00 Uhr im Konferenzraum (2.OG) des Arithmeums statt. Das Thema lautet 'Vertiefung in die Hypergraphen', alle interessierten Studenten sind herzlich eingeladen.