

Exercises 7

1) Given an undirected graph G , weights $w : E(G) \rightarrow \mathbb{R}_{\geq 0}$, $\mathcal{C} \subsetneq V(G)$ and $x : V(G) \setminus \mathcal{C} \rightarrow \{1, \dots, k\}$ for $k \in \mathbb{N}$. We are looking for positions $x : \mathcal{C} \rightarrow \mathbb{R}$ such that

$$\sum_{\{v,w\} \in E(G)} w(\{v,w\}) \cdot |x(v) - x(w)| \quad (1)$$

is minimized.

Prove:

- (a) There is always a placement x minimizing (1) with $x(v) \in \{1, \dots, k\}$ for all $v \in \mathcal{C}$.
- (b) A placement minimizing (1) can be found by solving $k-1$ minimum weight s - t -cut problems in digraphs with $O(|V(G)|)$ vertices and $O(|E(G)|)$ edges.

Hint: Consider the digraphs G_j ($j \in \{1, \dots, k-1\}$) defined by

$$\begin{aligned} V(G_j) &:= \{s, t\} \cup \mathcal{C} \quad \text{and} \\ E(G_j) &:= \{\{s, v\} \mid \exists w \in V(G) \setminus \mathcal{C}, x(w) \leq j, \{v, w\} \in E(G)\} \cup \\ &\quad \{\{v, w\} \mid v, w \in \mathcal{C}, \{v, w\} \in E(G)\} \cup \\ &\quad \{\{v, t\} \mid \exists w \in V(G) \setminus \mathcal{C}, x(w) > j, \{v, w\} \in E(G)\}. \end{aligned}$$

(4 points)

2) Assume that there is an oracle which returns a feasible placement of minimum bounding box net length for any input net list $(\mathcal{C}', P', \gamma', \mathcal{N}')$ containing only two-terminal nets in $O(T(|\mathcal{N}'|, |\mathcal{C}'|))$ time for some $T : \mathbb{N}^2 \rightarrow \mathbb{N}$.

Show that such an oracle can be used to find a feasible placement minimizing the bounding box net length of a general net list $(\mathcal{C}, P, \gamma, \mathcal{N})$ in $O(T(|\mathcal{P}|, |\mathcal{C}|))$ time.

(4 points)

Deadline: June 8 before the lecture (12.15 pm).