Chip Design
Summer term 2010

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Exercises 4

1) For a finite set $T \subset \mathbb{R}^2$ we define $\mathrm{BB}(T) := \max_{(x,y)\in T} x - \min_{(x,y)\in T} x + \max_{(x,y)\in T} y - \min_{(x,y)\in T} y$. Moreover, let $\textsc{Steiner}(T)$ be the length of a shortest rectilinear Steiner tree for T, and let $\mathrm{MST}(T)$ be the length of a minimum spanning tree in the complete graph on T, where edge weights are l_1-distances.

(a) Prove that $\mathrm{BB}(T) \le \textsc{Steiner}(T) \le \mathrm{MST}(T)$ for all finite sets $T \subset \mathbb{R}^2$.

(b) Prove that $\textsc{Steiner}(T) \le \frac{3}{2}\mathrm{BB}(T)$ for all $T \subset \mathbb{R}^2$ with $|T| \le 5$.

(c) Show that there exists no $k \in \mathbb{R}$ with $\textsc{Steiner}(T) \le k \cdot \mathrm{BB}(T)$ for all finite sets $T \subset \mathbb{R}^2$.

(4 points)

2) Let $T \subset \mathbb{R}^2$ be a finite set, $(x,y) \in T$ and $x' \in \mathbb{R}$ such that $\{(x'', y'') \in T : x'' < x'\} = \{(x,y)\}$. Show that there exists a shortest rectilinear Steiner tree for T which consists of a shortest rectilinear Steiner tree for $(T \setminus \{(x,y)\}) \cup \{(x',y)\}$ plus the edge $\{(x,y),(x',y)\}$.

(2 points)

3) Let $z_{\max}$ be a fixed odd natural number. As in the lecture, planes $1, 3, \ldots, z_{\max}$ are routing planes and $2, 4, \ldots, z_{\max} - 1$ are via planes. Moreover, we have capacitance factors $\alpha_z, \beta_z \in \mathbb{R}_{>0} \cup \{\infty\}$ for $z \in \{1, 3, \ldots, z_{\max}\}$ and $\gamma_{z,z'} = \gamma_{z',z} \in \mathbb{R}_{>0}$ for $z, z' \in \{1, 3, \ldots, z_{\max}\}$ and $|z - z'| = 2$.
Let $T \subset \mathbb{R}^2 \times \{1, 3, \ldots, z_{\max}\}$ be a finite set of pins and $T|_{\mathbb{R}^2} := \{(x, y) | (x, y, z) \in T$ for some $z\}$. Let $C := (V, E, m)$ be a connection for T (the wire models m are irrelevant here).
The capacitance of C is defined as

$$\sum_{\{(x,y,z),(x',y',z')\} \in E} \left(\alpha_z \cdot |x - x'| + \beta_z \cdot |y - y'| + \gamma_{z,z'} \cdot |z - z'| \right).$$

(a) Show that the planar projection of a minimum capacitance connection for T is not always a shortest rectilinear Steiner tree for $T|_{\mathbb{R}^2}$, even if all pins are on plane 1 and $\min_{z=1,3,\ldots,z_{\max}} \alpha_z = \min_{z=1,3,\ldots,z_{\max}} \beta_z$.

(b) Let Y be a rectilinear Steiner tree for $T|_{\mathbb{R}^2}$. We are looking for connections for T realizing Y, i.e. connections $C = (V, E, m)$ for T such that for all $\{(x, y), (x', y')\} \in E(Y)$ there exists exactly one $z \in \{1, 3, \ldots, z_{\max}\}$ with $\{(x, y, z), (x', y', z)\} \in E$ and for all $\{(x, y, z), (x', y', z')\} \in E$ we have $z \neq z'$ or $\{(x, y), (x', y')\} \in E(Y)$.

Describe a polynomial-time algorithm that computes a connection C for T realizing Y such that C has minimum capacitance.

What running time can you achieve?

(6 points)

Deadline: May 11 before the lecture (12.15 pm).