

Exercises 3

1) Implement the $O(n^3)$ -time algorithm from exercise 2.1 to compute the bounding box net length for a set of sets of rectangles.

The implementation must be done either in the C++ or C programming language respecting the C/C++ standard from 1999. You can easily achieve this by using the GNU-compiler (gcc or g++) and by including only standard headers (including the STL).

The input should be read either from an input pipe or directly from a file. The input format is as follows. The first line contains a number $n \in \mathbb{N}$ specifying the number of sets of rectangles. Then n blocks are following, each containing the information for one set. The first line of each block contains the number of rectangles in the corresponding set. The next k lines contain the four coordinates of one rectangle:

$x_{\min} y_{\min} x_{\max} y_{\max}$

The values define the rectangle $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}] \subset \mathbb{R}^2$. Note that the rectangles might be 2-, 1- or 0-dimensional and might overlap other rectangles.

In the following example there are two sets of rectangles, the first one containing three, the second one two rectangles:

```
2
3
0 0 3 4
3 3 5 4
1 2 5 2
2
4 0 6 1
3 5 4 5
```

The program should write the result (coordinates of a bounding box rectangle and the bounding box net length) to the standard output. Here for example:

```
bounding box: 4 1 4 2
bounding box netlength: 1
```

More examples can be found on the web page of the exercises class.

(12 points)

2) Let r be a pin and $\alpha > 0$.

- If r is a logical source, replace each signal $(a, s, \zeta, \xi) \in \Sigma(r)$ by $(a + \alpha, s, \zeta, \xi)$, i.e. all signals start α later.
- If r is no logical source, replace $delay_{(p,r),z}^{late}(\xi, s)$ by $delay_{(p,r),z}^{late}(\xi, s) + \alpha$ for all $p \in \delta^-(r)$, z, ξ and s , i.e. all signals at r are additionally delayed by α .

Let $slack^{late}(r)$ be the slack at r according to the original information. For a logical sink q denote by $\Sigma(q)$ and $\Sigma'(q)$ the signals at q calculated using the original and the modified delays, respectively.

Prove that for each logical sink q and all $(a', s, \zeta, \xi) \in \Sigma'(q)$ there exists an $a \in \mathbb{R}$ with $(a, s, \zeta, \xi) \in \Sigma(q)$ and

$$a' \leq \max\{a, rat^{late}(q, s, \zeta, \xi)\}$$

if and only if $\alpha \leq slack^{late}(r)$.

(4 points)

3) Let Y be a Steiner tree for a terminal set T in which all leaves are terminals. Prove:

- $|\{v \in V(Y) \setminus T : |\delta_Y(v)| > 2\}| \leq |T| - 2$.
- $\sum_{v \in T} (|\delta_Y(v)| - 1) = k - 1$, where k is the number of full components in T .

(4 points)

4) Let T be an instance of the RECTILINEAR STEINER TREE PROBLEM and $r \in T$. For a rectilinear Steiner tree Y for T we denote by $l(Y)$ the maximum length of a path from r to an element of $T \setminus \{r\}$ in Y .

- Describe an instance in which no shortest Steiner tree minimizes $l(Y)$ and no Steiner tree minimizing $l(Y)$ is shortest.
- Consider the problem of finding a shortest Steiner tree for which $l(Y)$ is minimum among all shortest Steiner trees. Is there always a tree with these properties which is a subgraph of the Hanan grid?

(4 points)

Deadline for exercise 1: May 16 (by e-mail to massberg@or.uni-bonn.de).

Deadline for exercises 2-3: May 4 before the lecture (12.15 pm).