Chip Design
Summer term 2010

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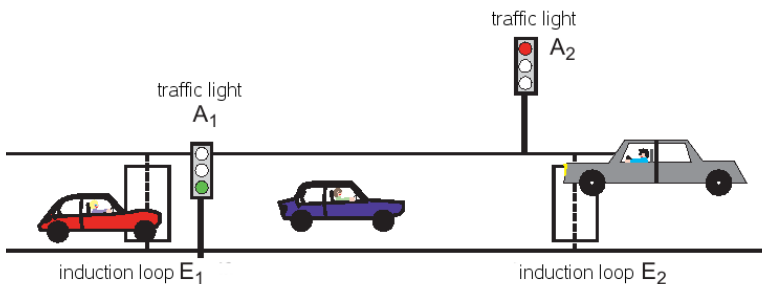
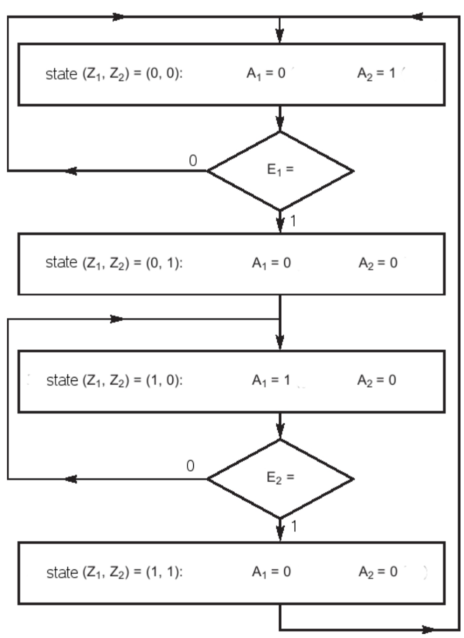
Exercises 1

1) We want to design a chip controlling two traffic lights (A_1, A_2) at a construction site on a street. Traffic light i is red if $A_i = 0$ and green if $A_i = 1$ ($i = 1, 2$). At most one of the traffic lights shall be green at the same time.
In front of each traffic light there is an induction loop (E_1, E_2) indicating whether a car is waiting ($= 1$) or not ($= 0$).



Moreover, there are four states, represented by two bits Z_1, Z_2. The following flow chart specifies how the states and A_1, A_2 shall change depending on the previous state (Z_1, Z_2) and on E_1, E_2:



Write A_1, A_2, Z_1, Z_2 as Boolean functions with variables Z_1, Z_2, E_1 and E_2. Construct a netlist realizing this logical description using a library containing inverters, *AND*s, and *OR*s, and draw it.

(4 points)

2) Replace the netlist you have found in Exercise 1 by a logically equivalent netlist only containing *NAND*s.

(4 points)

3) Prove that for every netlist with technology mapping there is a logically equivalent one that only contains *NAND*s.

(4 points)

4) Let $n \in \mathbb{N}$, $n \geq 7$. Let A_n be the set of Boolean functions $f : \{0,1\}^n \to \{0,1\}$ and $B_n := \{f \in A_n |$ there is a netlist realizing f with at most $\frac{2^{n-1}}{n}$ circuits, each with at most two inputs$\}$. Prove that

$$\frac{|B_n|}{|A_n|} < \frac{1}{n^{\frac{2^{n-1}}{n}}}.$$

(4 points)

Deadline: April 20 before the lecture (12.15 pm).