1) We want to design a chip controlling two traffic lights \((A_1, A_2)\) at a construction site on a street. Traffic light \(i\) is red if \(A_i = 0\) and green if \(A_i = 1\) \((i = 1, 2)\). At most one of the traffic lights shall be green at the same time. In front of each traffic light there is an induction loop \((E_1, E_2)\) indicating whether a car is waiting (= 1) or not (= 0).

Moreover, there are four states, represented by two bits \(Z_1, Z_2\). The following flowchart specifies how the states and \(A_1, A_2\) shall change depending on the previous state \((Z_1, Z_2)\) and on \(E_1, E_2\):

Write \(A_1, A_2, Z_1, Z_2\) as Boolean functions with variables \(Z_1, Z_2, E_1\) and \(E_2\). Construct a netlist realizing this logical description using a library containing inverters, ANDs, and ORs, and draw it.

(4 points)
2) Replace the netlist you have found in Exercise 1 by a logically equivalent netlist only containing NANDs. 

(4 points)

3) Prove that for every netlist with technology mapping there is a logically equivalent one that only contains NANDs. 

(4 points)

4) Let \( n \in \mathbb{N}, n \geq 7 \). Let \( A_n \) be the set of Boolean functions \( f : \{0, 1\}^n \to \{0, 1\} \) and \( B_n := \{ f \in A_n \mid \text{there is a netlist realizing } f \text{ with at most } \frac{2^{n-1}}{n} \text{ circuits, each with at most two inputs} \} \). Prove that

\[
\frac{|B_n|}{|A_n|} < \frac{1}{n^{\frac{2^{n-1}}{n}}}. 
\]

(4 points)

Deadline: April 20 before the lecture (12.15 pm).