

## Exercise Set 11

### Exercise 1:

Describe an exact algorithm for the TSP using the following technique: If the vertices are numbered from 1 to  $n$  we denote by  $\gamma(A, x)$  the length of a shortest 1- $x$ -path  $P$  with  $V(P) = A \cup \{1\}$  for all  $A \subseteq \{2, \dots, n\}$  and  $x \in A$ . The idea is to compute all these numbers. What running time can be achieved in contrast to the naive enumeration of all tours?

(4 points)

### Exercise 2:

The ANOTHER HAMILTONIAN CIRCUIT problem is the following: Given a graph  $G$  and a Hamiltonian circuit in  $G$ , is there another Hamiltonian circuit in  $G$ ?

- (i) Find a graph  $G = (V, E)$  with  $v_1, v_2, w_1, w_2 \in V$  that contains Hamiltonian  $v_1$ - $w_1$ - and  $v_2$ - $w_2$ -paths but no Hamiltonian  $v_1$ - $w_2$ -,  $v_1$ - $v_2$ -,  $w_1$ - $w_2$ - or  $v_2$ - $w_1$ -paths.
- (ii) Use the gadget from (i) to show that ANOTHER HAMILTONIAN CIRCUIT is  $NP$ -complete.
- (iii) Show that every edge in a 3-regular graph is contained in an even number of Hamiltonian circuits.
- (iv) Is ANOTHER HAMILTONIAN CIRCUIT for 3-regular graphs in  $P$ ?

(2+3+4+2 points)

### Special topic:

The next meeting of the institute's group of mentors takes place on Tuesday, July 13<sup>th</sup>, at 6:00 pm in the conference room of the Arithmeum. The topic is "Rectangle Packing" and all interested students are invited.

Please return the exercises until Tuesday, **July 13th, at 2:15 pm.**