Exercise Set 7

Exercise 1:
An approximation algorithm with absolute error $B$ is an algorithm $A$ such that $|A(I) - OPT| \leq B$ holds for any instance $I$.

(i) Prove: If $P \neq NP$, then for all $B \in \N$ there is no approximation algorithm with absolute error $B$ for the Steiner Tree Problem.

(ii) A construction similar to the one in (i) works for many problems, but there are also NP-hard problems which have approximation algorithms with absolute error 1. Find an NP-hard optimization variant of Partition for which such an algorithm exists (and prove that it exists).

(4+4 points)

Exercise 2:
Consider the Steiner Tree Problem on a network $N = (V, E, c, K)$ where $(V, E)$ is the underlying graph, $c : E \to \R_+$ describes the lengths of the edges, and $K \subseteq V$ is the terminal set. Let the complete distance network $N_D = (K, E_D, c_D)$ be a complete graph $(K, E_D)$ on the terminal set with edge lengths $c_D : E_D \to \R_+$ defined as $c_D(\{v, w\}) := \text{dist}_N(v, w)$, where $\text{dist}_N(v, w)$ is the length of a shortest path from $v$ to $w$ in $N$.

(i) Prove: If $T$ is a minimum spanning tree in $N_D$, then $c_D(T) \leq (2 - \frac{2}{|K|}) \cdot \text{smt}(N)$, where $\text{smt}(N)$ is the length of a minimum Steiner tree in $N$. (It follows that the algorithm of Kou, Markowsky, and Berman is a $(2 - \frac{2}{|K|})$-approximation.)

(ii) Show that the bound in (i) is best possible.

(4+3 points)

Special topic:
The next meeting of the institute’s group of mentors takes place on Tuesday, June 15th, at 6:00 pm in the conference room of the Arithmeum. The topic is “Hypergraphs Reloaded” and all interested students are invited.

Please return the exercises until Tuesday, June 15th, at 2:15 pm.