Exercise Set 2

Exercise 1:
Show that an otherwise polynomial-time algorithm that makes at most a constant number of calls to polynomial-time subroutines runs in polynomial time, but that a polynomial number of calls to polynomial-time subroutines may result in an exponential-time algorithm. (3 points)

Exercise 2:
Prove the $NP$-completeness of the following problems:

(i) Instance: An instance of 3SAT.
    Task: Is there a truth assignment that makes at least one literal true and at least one literal false in each clause.

(ii) Instance: An undirected graph $G = (V, E)$ and an integer $k$.
    Task: Is there an $X \subseteq V$ with $|X| \leq k$ such that $|\delta(X)| \geq k$?

Hint: Use (i) to prove (ii). (3+4 Points)

Exercise 3:
Prove the $NP$-completeness of the following problem: Given a directed graph $G = (V, E)$ and an integer $k$, is there an $X \subseteq V$ with $|X| \leq k$ such that every directed circuit in $G$ contains at least one node in $X$? (3 Points)

Exercise 4:
Prove the $NP$-completeness of the following problem: Given natural numbers $A$ and $a_i$ for $1 \leq i \leq n$, is there a disjoint axis-parallel packing of the $n$ squares with side lengths $a_i$ inside a rectangle with area $A$. (4 Points)

Please return the exercises until Tuesday, May 4th, at 2:15 pm.