Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2010

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Exercise Set 2

Exercise 1:
Show that an otherwise polynomial-time algorithm that makes at most a constant number of calls to polynomial-time subroutines runs in polynomial time, but that a polynomial number of calls to polynomial-time subroutines may result in an exponential-time algorithm.

(3 points)

Exercise 2:
Prove the NP-completeness of the following problems:

(i) Instance: An instance of 3SAT.
Task: Is there a truth assignment that makes at least one literal true and at least one literal false in each clause.

(ii) Instance: An undirected graph $G = (V, E)$ and an integer k.
Task: Is there an $X \subseteq V$ with $|X| \leq k$ such that $|\delta(X)| \geq k$?

Hint: Use (i) to prove (ii).

(3+4 Points)

Exercise 3:
Prove the NP-completeness of the following problem: Given a directed graph $G = (V, E)$ and an integer k, is there an $X \subseteq V$ with $|X| \leq k$ such that every directed circuit in G contains at least one node in X?

(3 Points)

Exercise 4:
Prove the NP-completeness of the following problem: Given natural numbers A and a_i for $1 \leq i \leq n$, is there a disjoint axis-parallel packing of the n squares with side lengths a_i inside a rectangle with area A.

(4 Points)

Please return the exercises until Tuesday, **May 4th, at 2:15 pm**.