Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2010

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Exercise Set 1

Exercise 1:
Let $f(n)$ and $g(n)$ be any two of the following functions. For each pair, determine whether $f(n) = \mathcal{O}(g(n))$ or $f(n) = \Omega(g(n))$ or $f(n) = \Theta(g(n))$ holds:

(a) $(\log n)^n$ (b) $n^{\log n}$
(c) n^2 (d) n^2 if n is odd, n^n otherwise

(4 Points)

Exercise 2:
Describe a Turing machine which compares two strings. As an input, it should accept a string $a\#b$ with $a, b \in \{0,1\}^*$. The output should be 1 for $a = b$ and 0 otherwise.

(4 points)

Exercise 3:
Prove that SAT remains NP-complete if each clause contains exactly three literals and each variable is contained in at most four clauses.

(4 points)

Exercise 4:
Prove the NP-completeness of the following problem:

- INSTANCE: Natural numbers W and H and pairs $(w_i, h_i) \in \mathbb{N}^2$ for $1 \le i \le n$.
  TASK: Is there a disjoint axis-parallel packing of n rectangles having width w_i and height h_i inside a rectangle of width W and height H. Precisely, are there pairs $(x_i, y_i) \in \mathbb{N}^2$ for $1 \le i \le n$ such that

  - $R_i \subseteq (0, W) \times (0, H)$ for $1 \le i \le n$ and
  - $(x, y) \in \mathbb{R}^2 \to |\{i \mid (x, y) \in R_i\}| \le 1$,

  where $R_i := (x_i, x_i + w_i) \times (y_i, y_i + h_i)$.

(3 points)

Please return the exercises until Tuesday, **April 27th, at 2:15 pm**.