Exercise Set 6

Exercise 1:
Consider the Fractional MultiKnapsack problem: Given natural numbers $m$, $n$, $w_i$, $c_{ij}$, and $W_j$ for $1 \leq i \leq n$ and $1 \leq j \leq m$, find $x_{ij} \in [0, 1]$ satisfying $\sum_{j=1}^{m} x_{ij} = 1$ for all $1 \leq i \leq n$ and $\sum_{i=1}^{n} x_{ij} w_{ij} \leq W_j$ for all $1 \leq j \leq m$, such that $\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} c_{ij}$ is minimum. Provide a polynomial-time combinatorial algorithm for this problem or prove that it is $NP$-hard.

(4 Points)

Exercise 2:
Suppose that in an instance $a_1, \ldots, a_n$ of the Bin Packing problem we have $a_i > \frac{1}{3}$ for $1 \leq i \leq n$.

(i) Reduce the problem to the Cardinality Matching problem.

(ii) Describe a linear-time algorithm that solves the problem.

(3+2 Points)

Exercise 3:
An algorithm for the Bin Packing problem is called monotone if for inputs $S$ and $T$ with $S \subseteq T$ the algorithm needs at least as many bins for $T$ as for $S$. Prove:

(i) Next Fit is monotone.

(ii) First Fit is not monotone.

(3+3 Points)

Please return the exercises until Tuesday, May 26th, at 2:15 pm.