

## Exercise Set 4

**Exercise 1:**

Show that any 4-colourable graph with  $n$  vertices can be coloured with  $\mathcal{O}(n^{\frac{2}{3}})$  colours in polynomial time.

(4 points)

**Exercise 2:**

Describe an algorithm which decides if a graph  $G = (V, E)$  is 4-colourable with a running time of  $\mathcal{O}(|E| \cdot 2^{|V|})$ .

(4 points)

**Exercise 3:**

Show with a reduction from 3SAT that MAX-2-SAT is *NP*-hard.

(4 points)

**Exercise 4:**

Show that the CROSSWORD PUZZLE problem is *NP*-complete: Given an integer  $n$ , a subset  $B \subseteq \{1, \dots, n\}^2$  of black squares, and a finite dictionary  $D \subseteq \Sigma^*$ , decide if there is a mapping  $F : \{1, \dots, n\}^2 \setminus B \rightarrow \Sigma$  such that all maximal words  $(F(i, j), \dots, F(i, j+k))$  and  $(F(i, j), \dots, F(i+k, j))$  are in  $D$ .

(4 points)

Please return the exercises until Tuesday, **May 12th, at 2:15 pm.**