Prof. Dr. S. Hougardy
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Exercise Set 3

Exercise 1:
Formulate fast 2-factor approximation algorithms for the following problems and prove the approximation factor:

(i) Given an undirected graph $G = (V, E)$, what is the diameter of G? (The diameter of G is defined as $\text{diam}(G) := \max_{v,w \in V} \text{dist}(v, w)$, where $\text{dist}(v, w)$ is the length of a shortest v-w-path.)
Hint: Linear runtime is possible.

(ii) Given a directed graph with edge weights, find a directed acyclic subgraph of maximum weight.

(4+4 Points)

Exercise 2:
Consider the following greedy algorithm for VERTEX COVER: Start with $C = \varnothing$. While there are still edges in G, choose the node in G with the largest degree, add it to C, and delete it from G.

(i) Show that the algorithm never produces a solution which is more than $\log n$ times the optimum.

(ii) Find a family of graphs in which the $\log n$ bound is achieved in the limit.

(4+2 Points)

Exercise 3:
Show that an otherwise polynomial-time algorithm that makes at most a constant number of calls to polynomial-time subroutines runs in polynomial time, but that a polynomial number of calls to polynomial-time subroutines may result in an exponential-time algorithm.

(3 points)

Exercise 4:
Consider an optimization problem $\mathcal{P}$ and the corresponding decision problem $\mathcal{P}'$. Show that if $\mathcal{P}'$ can be solved in polynomial time, then $\mathcal{P}$ can also be solved in polynomial time.

(3 points)

Please return the exercises until Tuesday, **May 5th, at 2:15 pm**.