

## Exercise Set 1

### Exercise 1:

Show that for any polynomial  $p(n)$  and any constant  $c$  there is an integer  $n_0$  such that  $2^{cn} > p(n)$  holds for all  $n \geq n_0$ .

(3 Points)

### Exercise 2:

Let  $f(n)$  and  $g(n)$  be any two of the following functions. For each pair, determine whether  $f(n) = \mathcal{O}(g(n))$  or  $f(n) = \Omega(g(n))$  or  $f(n) = \Theta(g(n))$  holds:

- |                  |                  |  |
|------------------|------------------|--|
| (a) $n^2$        | (b) $n^2 \log n$ | (c) $2^n$                                |
| (d) $n^{\log n}$ | (e) $2^{2^n}$    | (f) $n^2$ if $n$ is odd, $2^n$ otherwise |

(4 Points)

### Exercise 3:

Describe a Turing machine which mirrors a string. As an input, it should accept a string  $a_1 a_2 \dots a_n$  with  $a_i \in \{0, 1\}$  for  $1 \leq i \leq n$ . The output should be the reverse string  $a_n a_{n-1} \dots a_1$ .

(4 points)

### Exercise 4:

Give a linear-time algorithm for SAT which computes a truth assignment where at least half of the clauses are satisfied.

(2 points)

### Exercise 5:

Prove that SAT remains *NP*-complete if each clause contains exactly three literals and each variable is contained in at most four clauses.

(4 points)

Please return the exercises until Tuesday, **April 21st, at 2:15 pm.**