Minimum Bounded Degree Spanning Trees

Refuting a Conjecture of M. X. Goemans

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Institute for Operations Research

March 1, 2016

The Minimum Bounded Degree Spanning Tree Problem

(MBDST Problem)

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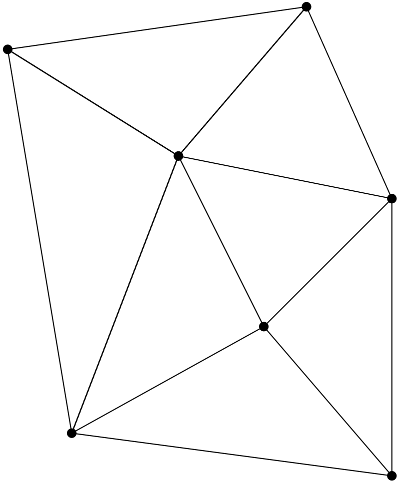
Input:

- graph $G = (V, E)$



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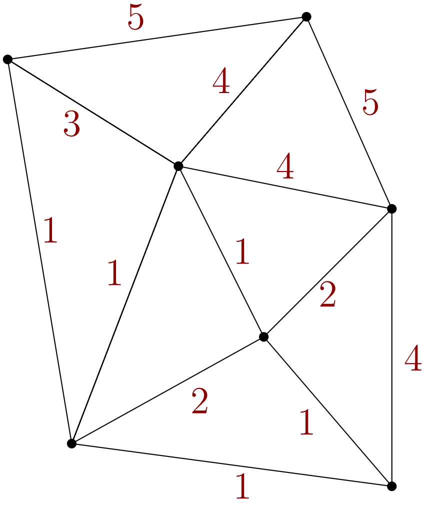

The Minimum Bounded Degree Spanning Tree Problem

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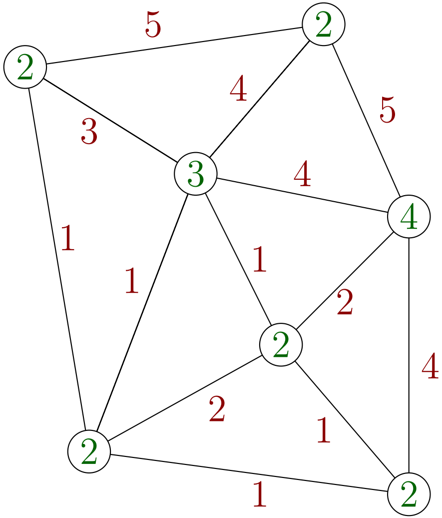

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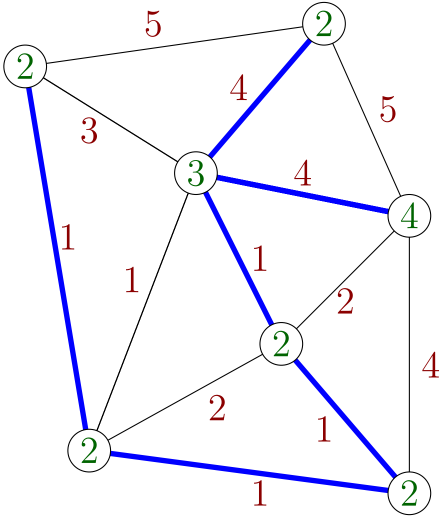
Output:
minimum cost spanning tree
satisfying the degree bounds



Goemans' Approximation Approach

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IP-formulation:

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\min & c^T x & & & \\
\text{s.t.} & x(E[S]) & \leqslant & |S| - 1 & \forall S \subseteq V,\ S \neq \emptyset \\
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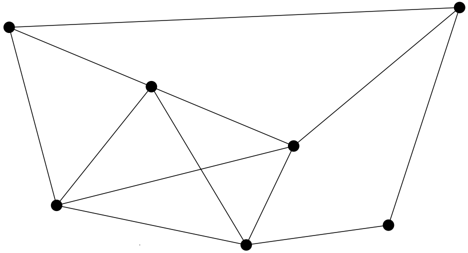

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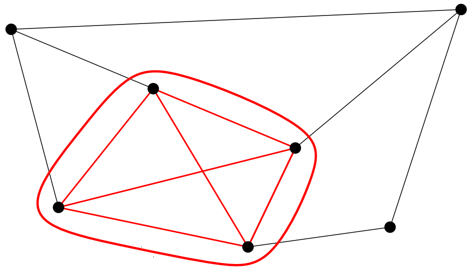

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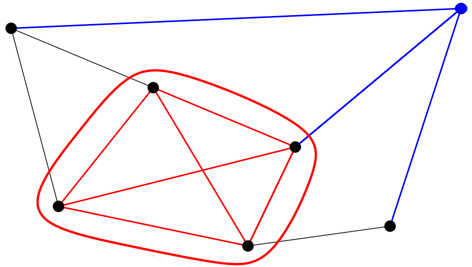

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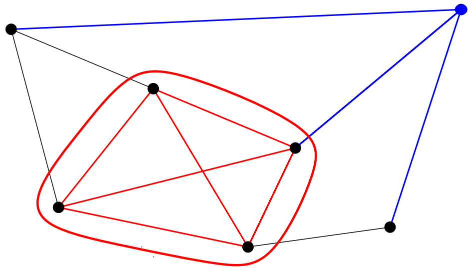

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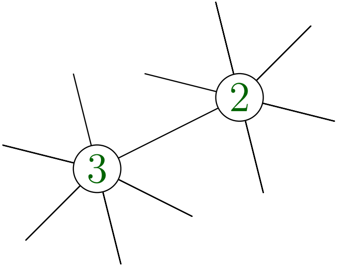

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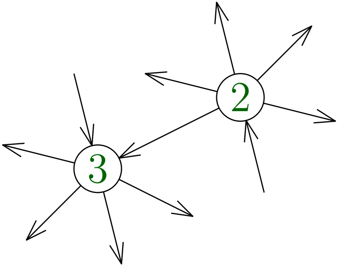

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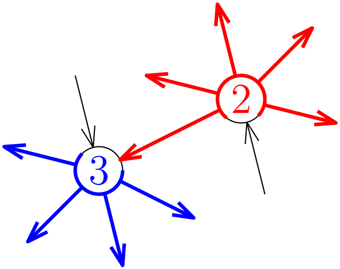

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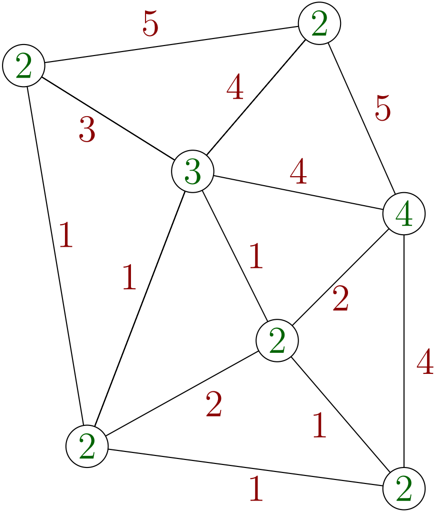
- solve LP-relaxation

$$x^* = \arg\min_{x \in P_{BDST}(G,d)} c^\top x$$



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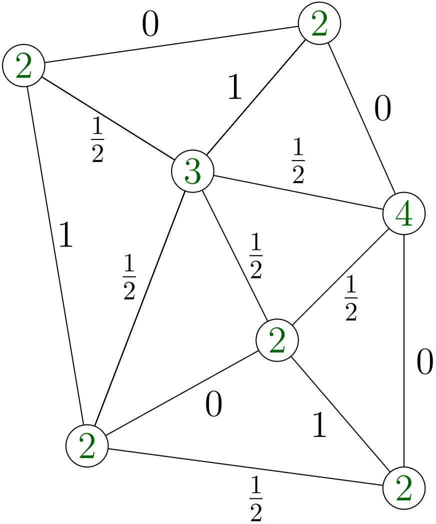

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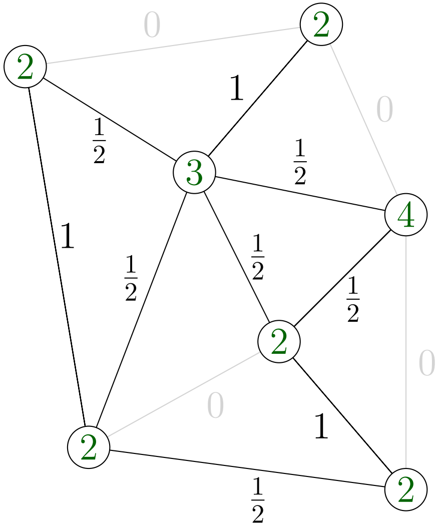

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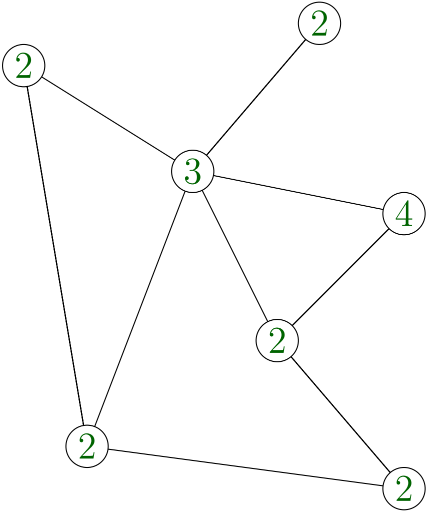

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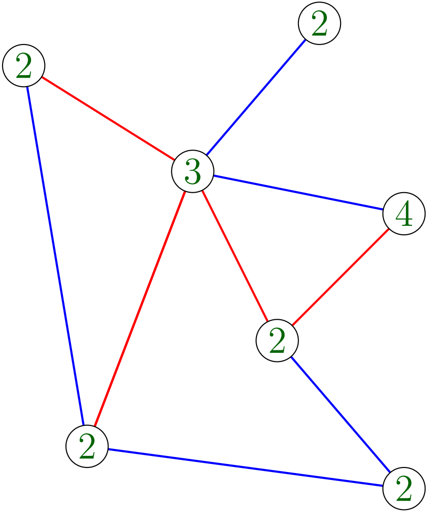

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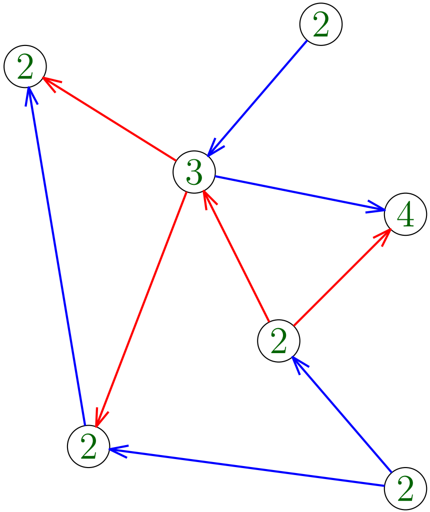
- orient towards leaves



Improving the Algorithm

Conjecture

For any extreme point x^* of $P_{BDST}(G, d)$ with support E^*, there exists an orientation A^* of E^* such that

$$\sum_{e \in \delta^-_{A^*}(v)} \Big(1 - x^*(e)\Big) \leqslant 1 \qquad \forall v \in V.$$

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- interpretation: orientation of spare $z^* := 1 - x^*$

Spare Orientation: An Example

- extreme point x^*
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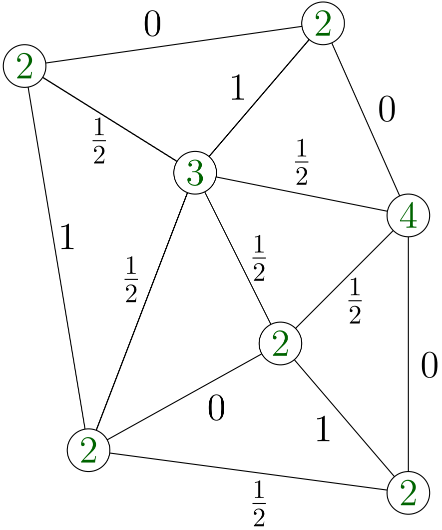

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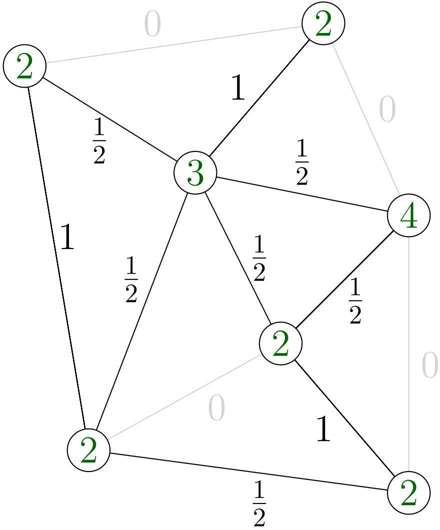

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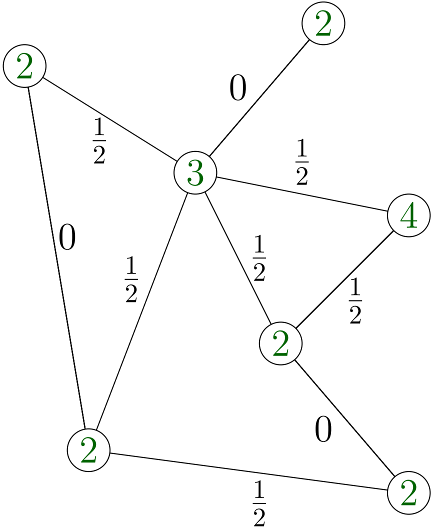

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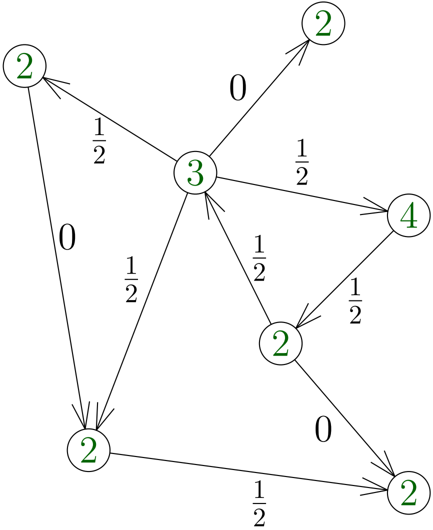

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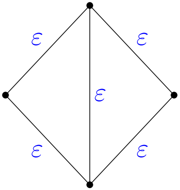
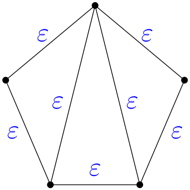


$\varepsilon < \frac{1}{5}$



$\varepsilon < \frac{2}{7}$

Constructing a Counterexample

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- non-integral degree bounds

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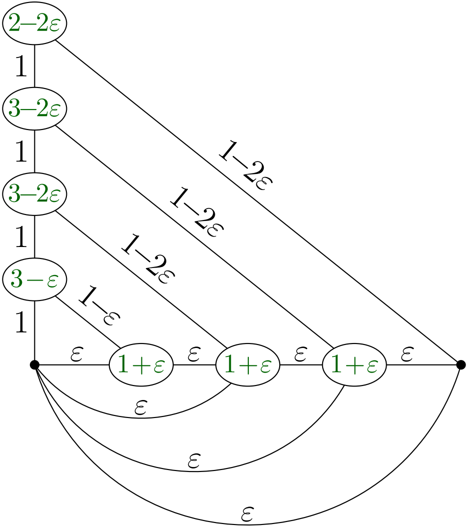
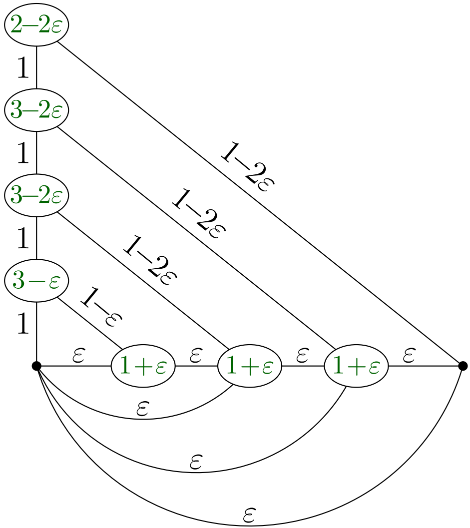
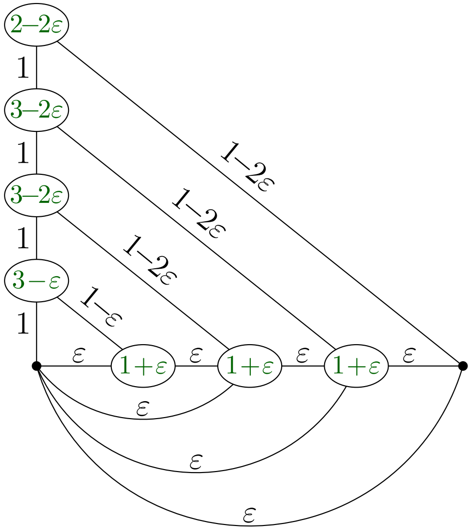


Checking Feasibility in $P_{BDST}(G, d)$



Checking Feasibility in $P_{BDST}(G, d)$

- decomposition as convex combination of spanning trees



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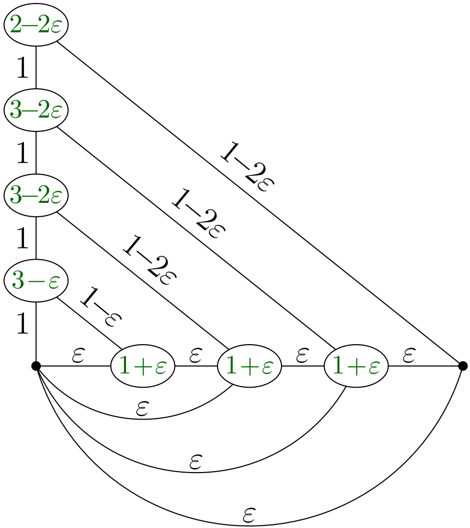
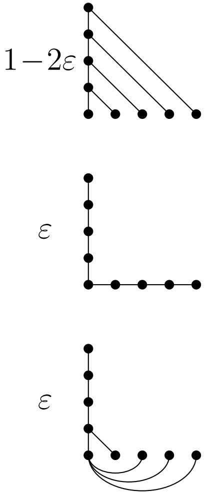
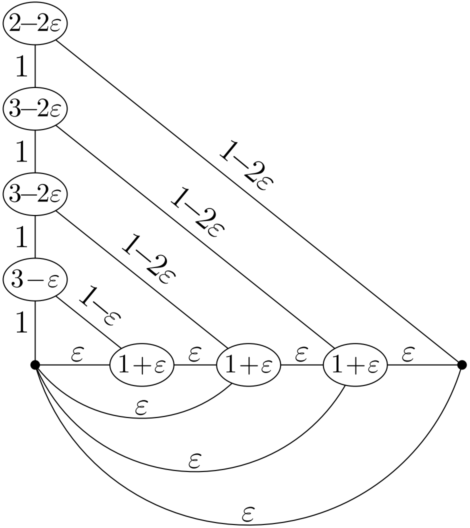




Tight Constraints form a Full-Rank System



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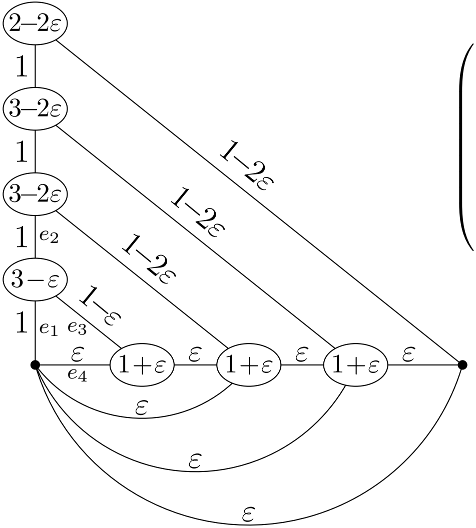

$$\begin{array}{c} \begin{matrix} e_1 & \quad e_2 & \quad e_3 & \quad e_4 & \quad \dots \end{matrix} \\ \begin{pmatrix} \quad & \quad & \quad & \quad & \quad \\ \\ \\ \\ \end{pmatrix} x = \begin{pmatrix} \quad \\ \\ \\ \\ \end{pmatrix} \end{array}$$

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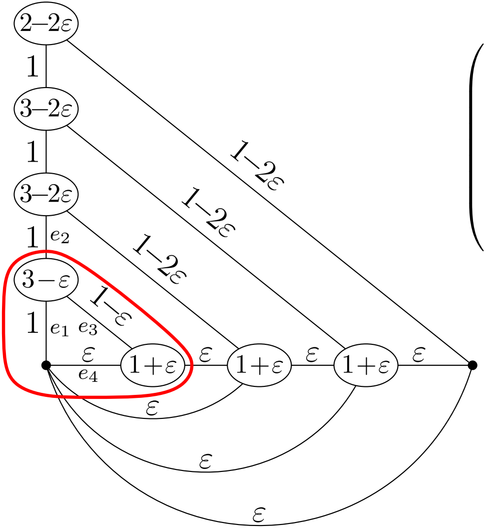

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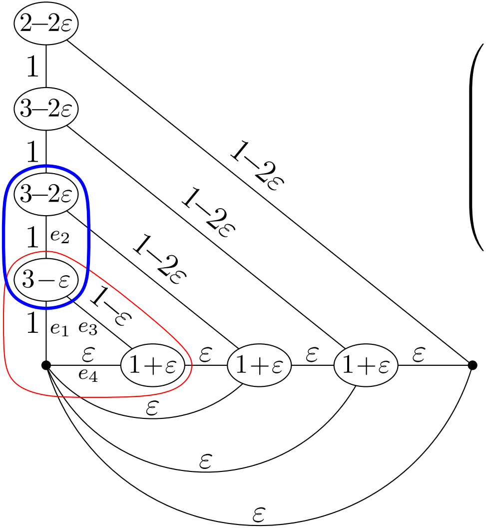

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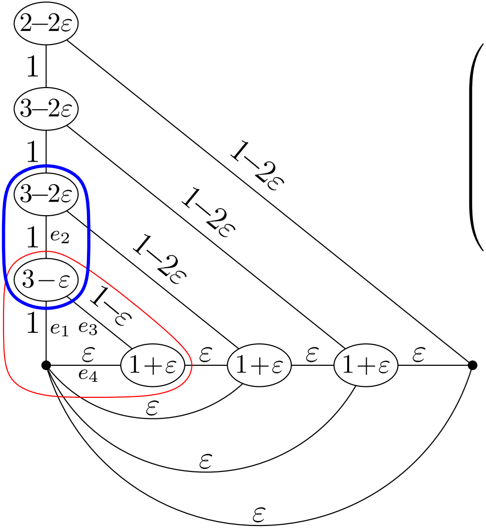
\begin{pmatrix} 1 & 0 & 1 & 1 & \ldots \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \ldots \\ & & & & \\ & & & & \end{pmatrix} x = \begin{pmatrix} 2 \\ \mathbf{1} \\ \\ \end{pmatrix}
$$

Tight Constraints form a Full-Rank System



$$
\begin{array}{ccccc}
e_1 & e_2 & e_3 & e_4 & \dots
\end{array}
$$

$$
\begin{pmatrix}
1 & 0 & 1 & 1 & \dots \\
0 & 1 & 0 & 0 & \dots \\
\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots \\
 & & & & \\
\end{pmatrix} x = \begin{pmatrix} 2 \\ 1 \\ \mathbf{1} \\ \\ \end{pmatrix}
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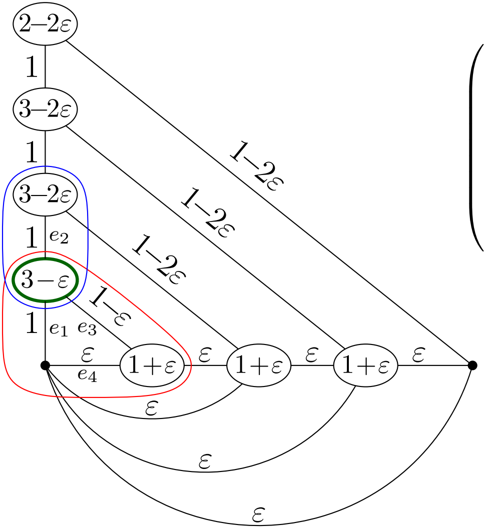

$$
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$$
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1 & 0 & 1 & 1 & \ldots \\
0 & 1 & 0 & 0 & \ldots \\
1 & 0 & 0 & 0 & \ldots \\
\mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \ldots \\
 & & & &
\end{pmatrix} x = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3-\varepsilon \\ \end{pmatrix}
$$

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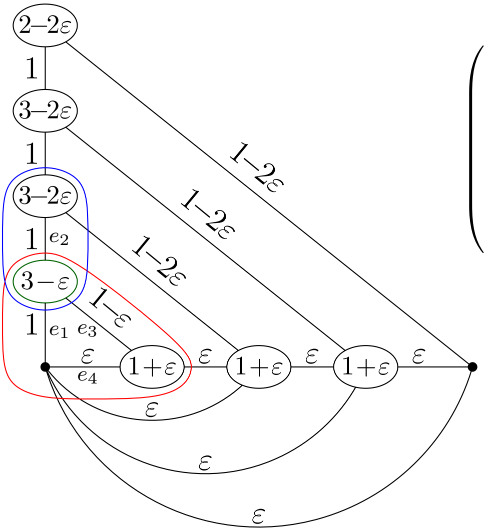

$$\begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & \ldots \end{array}$$
$$\begin{pmatrix} 1 & 0 & 1 & 1 & \ldots \\ 0 & 1 & 0 & 0 & \ldots \\ 1 & 0 & 0 & 0 & \ldots \\ 1 & 1 & 1 & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \end{pmatrix} x = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3-\varepsilon \\ \vdots \end{pmatrix}$$

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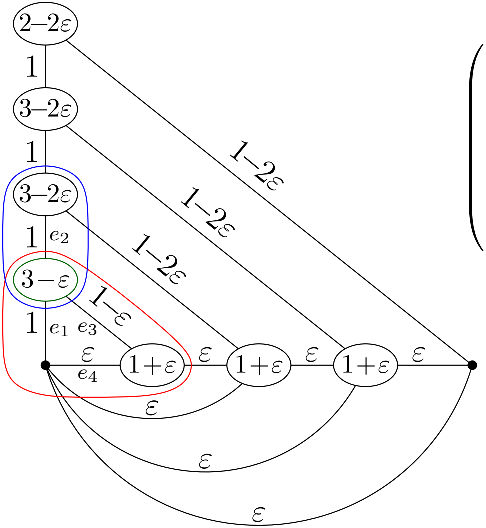

$$
\begin{array}{ccccc}
e_1 & e_2 & e_3 & e_4 & \dots
\end{array}
$$

$$
\begin{pmatrix}
1 & 0 & 1 & 1 & \dots \\
0 & 1 & 0 & 0 & \dots \\
1 & 0 & 0 & 0 & \dots \\
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\vdots & \vdots & \vdots & \vdots &
\end{pmatrix} x = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3-\varepsilon \\ \vdots \end{pmatrix}
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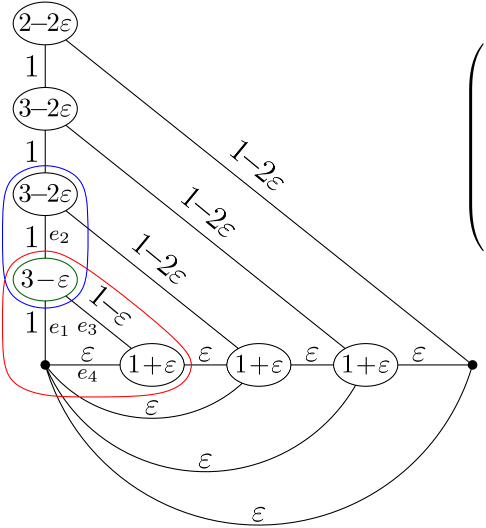

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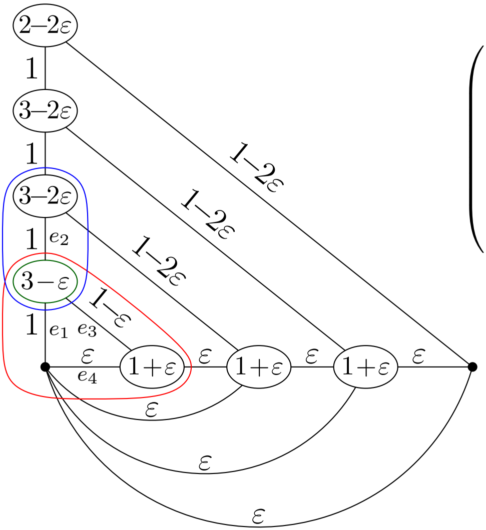
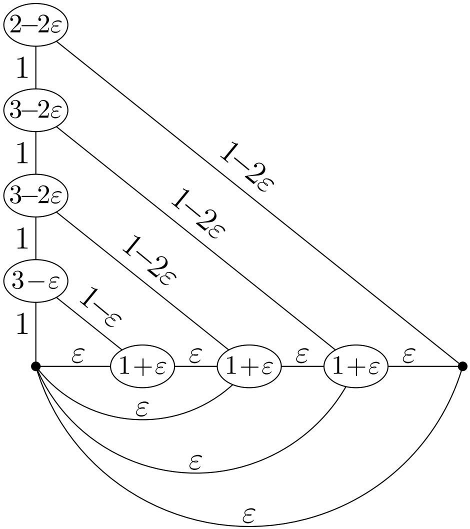


$$
\begin{array}{c}
\begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & \ldots \end{array} \\
\left(\begin{array}{ccccc}
1 & 0 & 1 & 1 & \ldots \\
0 & 1 & 0 & 0 & \ldots \\
1 & 0 & 0 & 0 & \ldots \\
1 & 1 & 1 & 0 & \ldots \\
\vdots & \vdots & \vdots & \vdots &
\end{array}\right) x = \left(\begin{array}{c} 2 \\ 1 \\ 1 \\ 3-\varepsilon \\ \vdots \end{array}\right) \\
\begin{array}{ccccc}
0 & 0 & 1 & 0 & \ldots \\
0 & 0 & 0 & 1 & \ldots \\
 & & \vdots & &
\end{array}
\end{array}
$$

How to Impose Fractional Degree Bounds?



Feasible Degree Bound Slacks

Definition

A number $r \in \mathbb{Q}_{\geqslant 0}$ is a *feasible degree bound slack* if there exists an integral instance (G, d), an extreme point $x \in P_{BDST}(G, d)$ and a vertex v such that $d(v) - x(\delta(v)) = r$.

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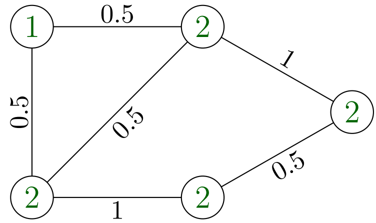

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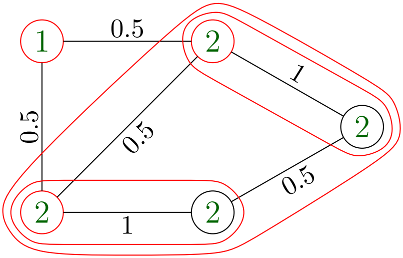

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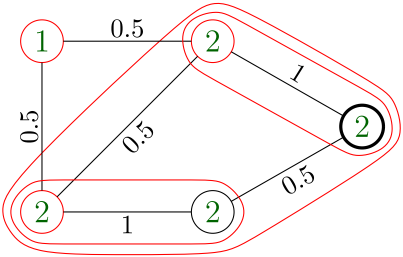
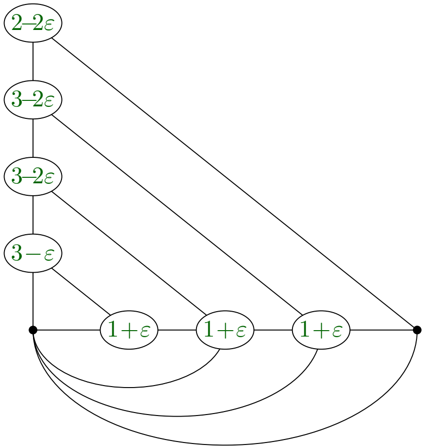


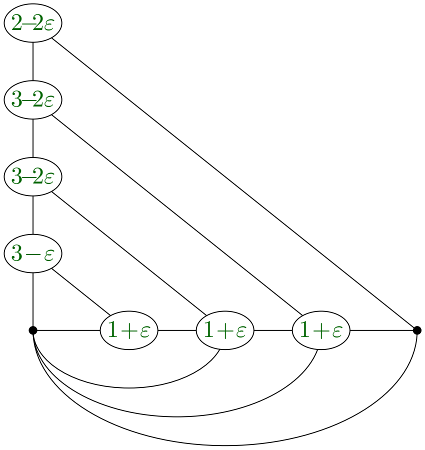
$\Longrightarrow$ $\frac{1}{2}$ is a feasible degree bound slack

Modelling Fractional Degree Bounds



Modelling Fractional Degree Bounds

$$\varepsilon = \frac{1}{4}$$



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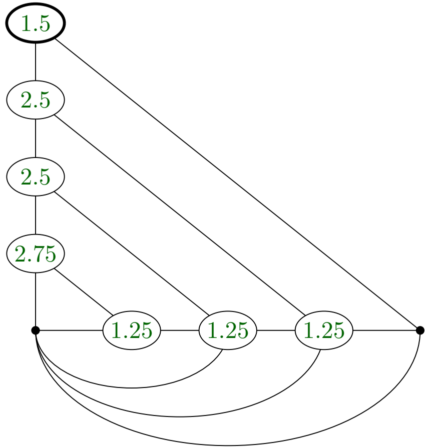

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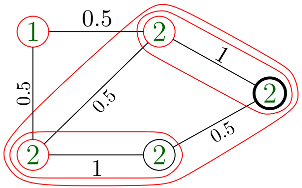

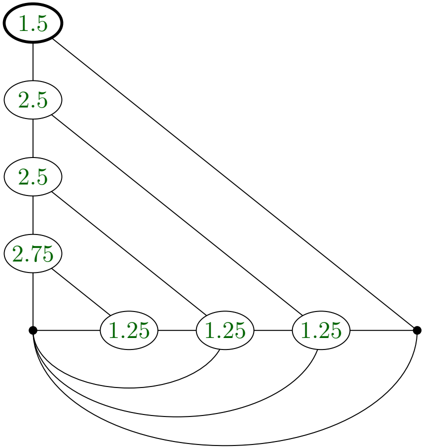

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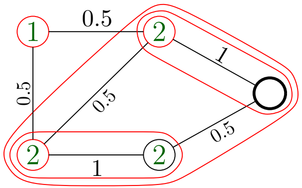

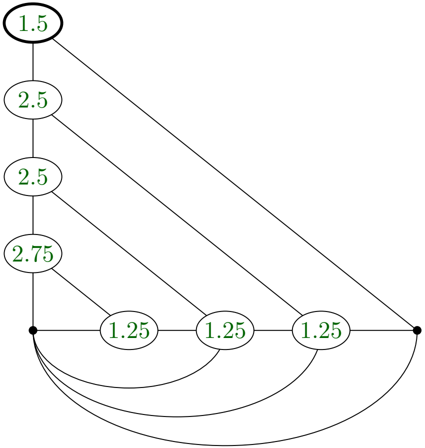

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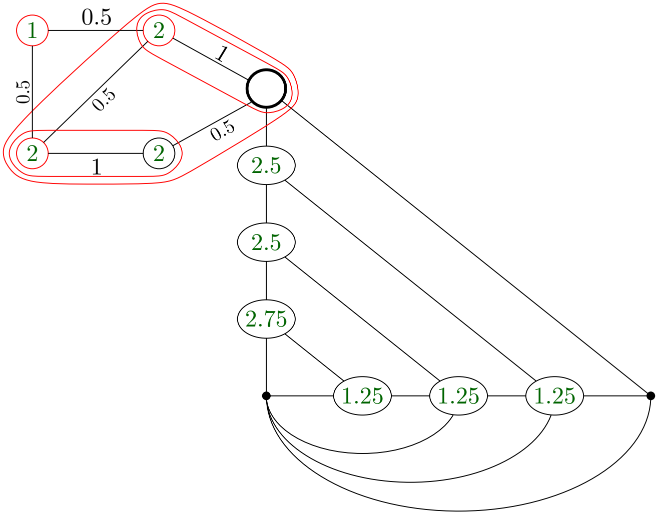

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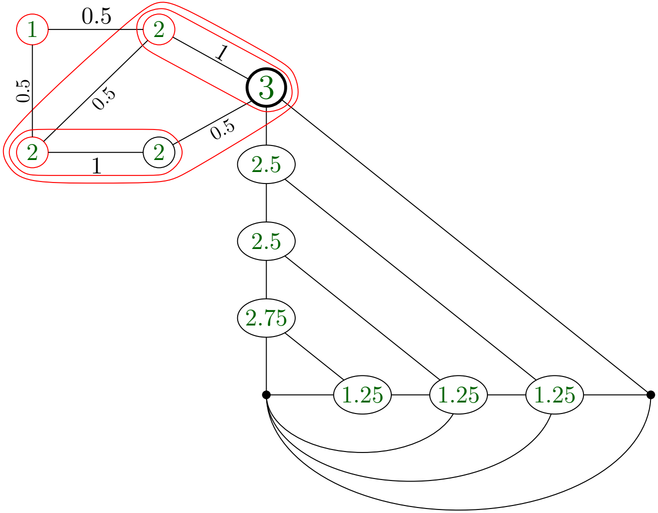

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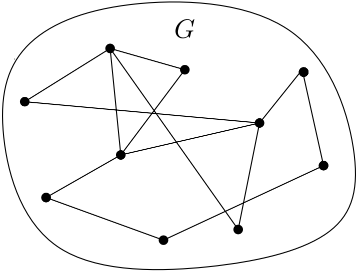

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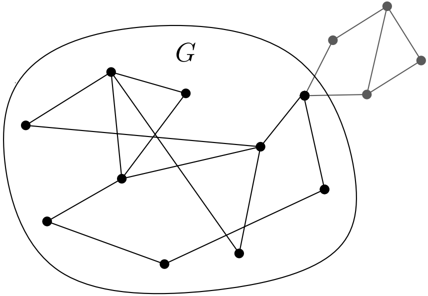

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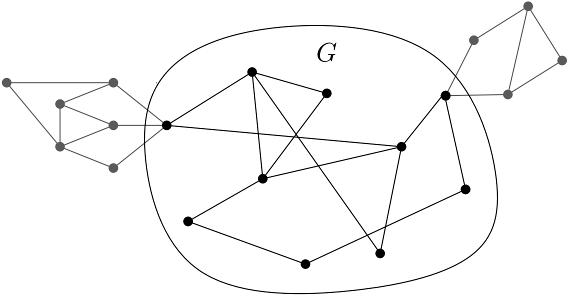

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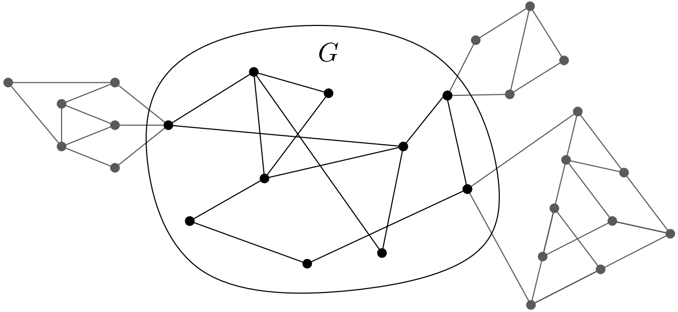

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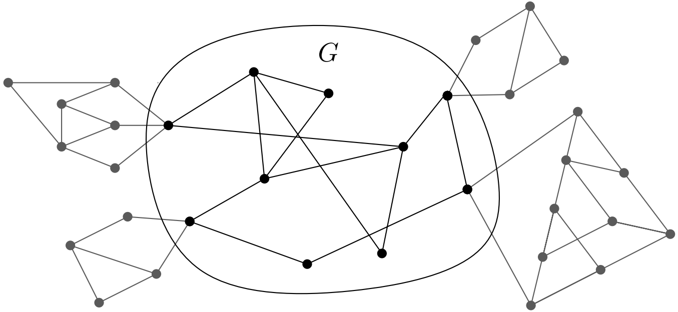

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Theorem

Every $r \in \mathbb{Q}_{\geqslant 0}$ *is a feasible degree bound slack.*

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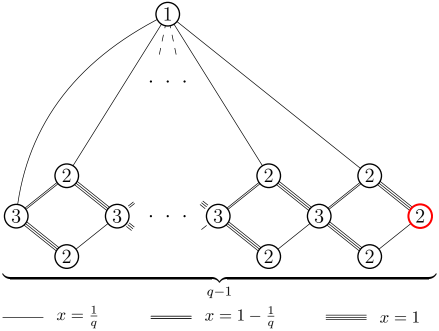
Lemma 1

For $q \geqslant 2$, $\frac{q-1}{q}$ is feasible.



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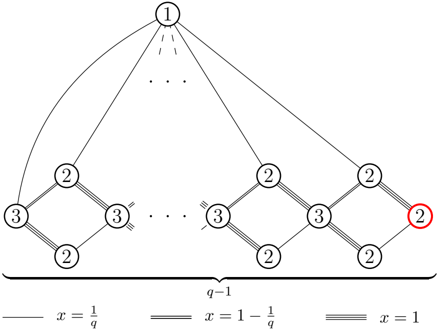
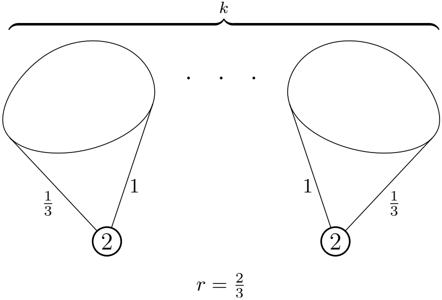


Lemma 2

If r is feasible, then so is $k\,r$ for all $k \in \mathbb{Z}$.



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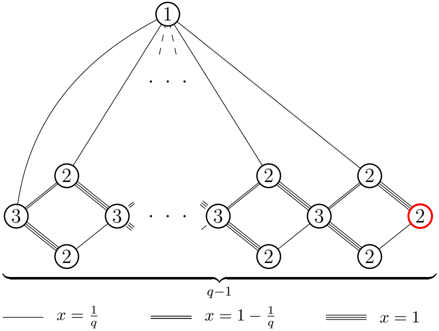

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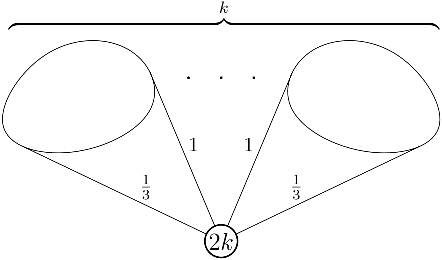
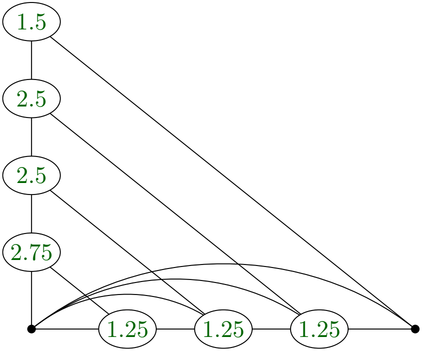


$r = \frac{2}{3}$

Conclusion



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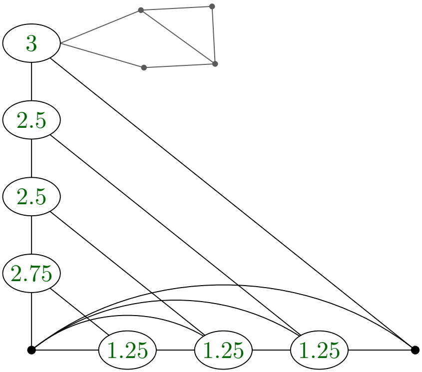

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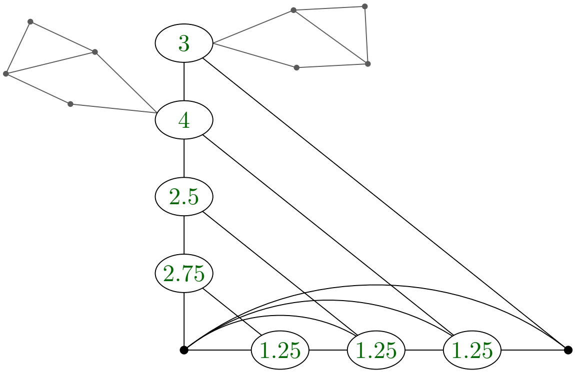

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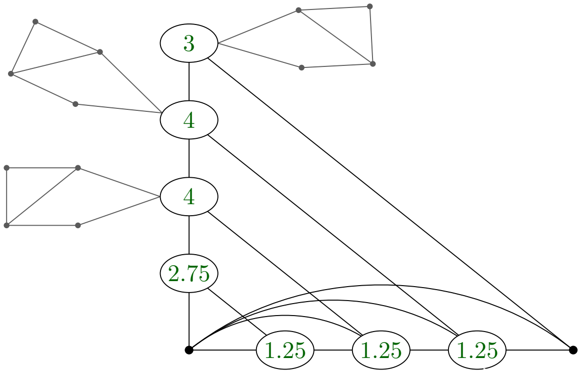

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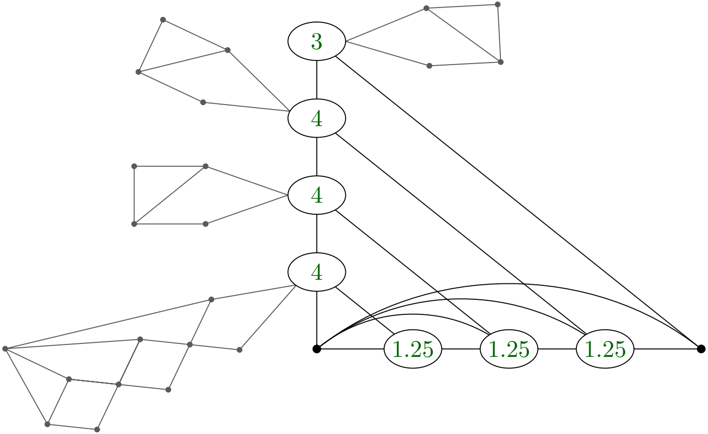

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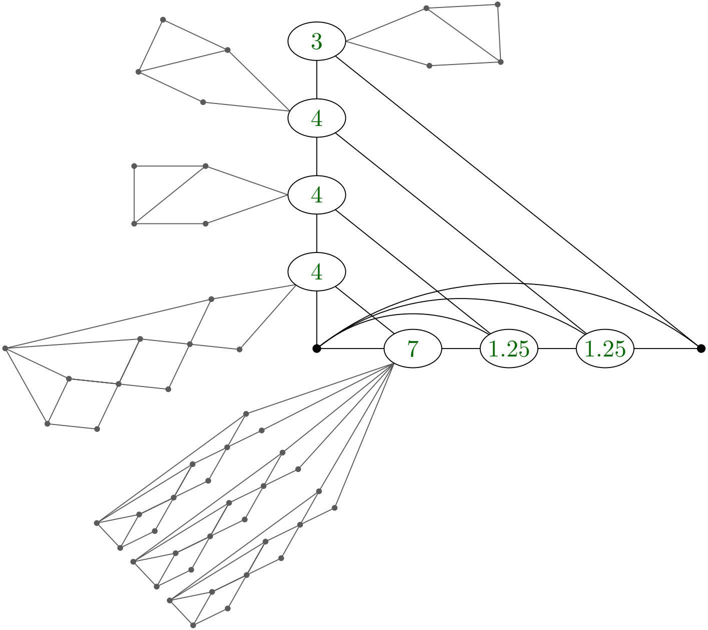

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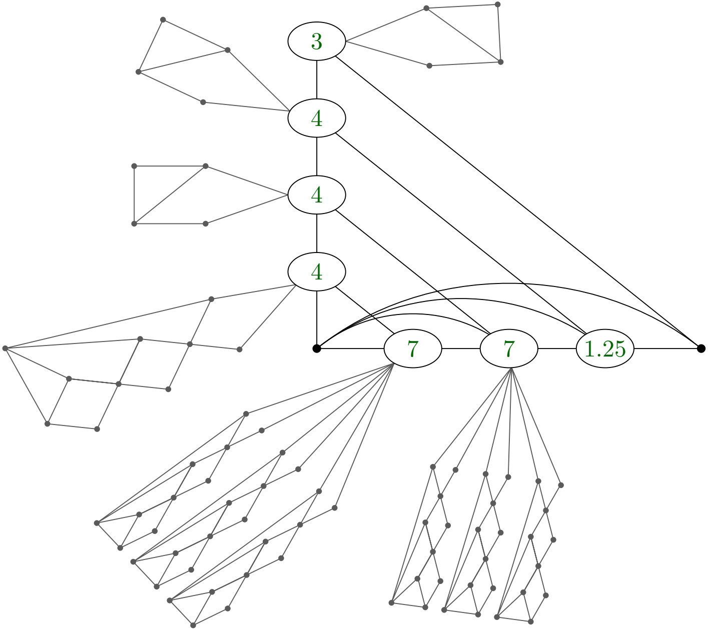

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