Winter semester 2013/14 Research Institute for Discrete Mathematics
Prof. Dr. S. Held University of Bonn

Linear and Integer Optimization
Exercise Sheet 13

Exercise 13.1: Let P the convex hull of the three points $(0,0)$, $(1,0)$, and $(\frac{1}{2},k)$ in $\mathbb{R}^2$, where $k \in \mathbb{N}$. Prove that $P^{(2k-1)} \neq P_I$, but $P^{(2k)} = P_I$. (5 Points)

Exercise 13.2: Let $P \subseteq [0,1]^n$ be a polytope in the unit hypercube with $P_I = \emptyset$. Prove that $P^{(n)} = \emptyset$. (5 Points)

Exercise 13.3: Consider the following facility location problem. We are given a set of n customers with demands $d_1, \ldots, d_n$, and m optional facilities. Each facility $i = 1 \ldots, m$ can be opened at cost f_i. It has a capacity of u_i and a distance c_{ij} to each customer $j = 1, \ldots, n$. The task is to open a set of facilities and to connect each customer to an open facility such that the total demand of the customers assigned to a facility does not exceed its capacity. The objective is to minimize the total facility opening cost plus the total connection cost. The problem can be formulated as the following integer program:

$$\min \left\{ \sum_{i.j} c_{ij} x_{ij} + \sum_i f_i y_i : \sum_j d_j x_{ij} \leq u_i y_i \ \forall i, \sum_i x_{ij} = 1 \ \forall j, x_{ij}, y_i \in \{0,1\} \right\}.$$

Apply Lagrangean relaxation in two ways:

1. relaxing $\sum_j d_j x_{ij} \leq u_i y_i$ for all i,
2. relaxing $\sum_i x_{ij} = 1$ for all j.

Which Lagrangean dual yields a tighter bound? (5 Points)

Exercise 13.4: Prove: If a linear program $\max\{c^\intercal x : Ax \leq b; x \geq 0\}$ has feasible solutions and the set of feasible solutions is bounded, then there is a strictly feasible dual solution $y, z > 0$ with $A^T y - z = c$.
Use this result to proof Corollary 9.5 in the lecture: If a primal feasible set has a non-emtpy interior and is bounded, then for each $\mu > 0$ there exists a unique solution to

$$\begin{aligned} Ax + w &= b, \\ A^\intercal y - z &= c, \\ XZ &= \mu \mathbb{1}, \\ WY &= \mu \mathbb{1}. \end{aligned}$$

(5 Points)

Submission deadline: Tuesday, 28.1.2014, before the lecture.