Winter semester 2013/14 Research Institute for Discrete Mathematics
Prof. Dr. S. Held University of Bonn

Linear and Integer Optimization

Exercise Sheet 12

Exercise 12.1: Prove that the system

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

is not TDI. Furthermore, give a TDI-system representing the same polyhedron (including a proof that it is TDI).

(5 Points)

Exercise 12.2: Let $a \neq 0$ be an integral vector and β a rational number. Prove that the inequality $a^\intercal x \leq \beta$ is TDI if and only if the components of a are relatively prime. (5 Points)

Exercise 12.3: Let $A \in \{0,1\}^{m \times n}$ be a matrix where in each column the 1's are arranged consecutively, i.e. for each column $j \in \{1, \dots, n\}$ there are $i_1^j, i_2^j \in \{1, \dots, m\}$ s.t.:

$$a_{ij} = \begin{cases} 1, & i_1^j \leq i \leq i_2^j \\ 0, & \text{else} \end{cases}$$

for $j \in \{1, \dots, n\}$ and $i \in \{1, \dots, m\}$ (if $i_1^j > i_2^j$, the column consists of zeros only). Show that A is totally unimodular. (5 Points)

Exercise 12.4:
Show that $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ is not totally unimodular, but $\{x : Ax = b\}$ is integral for all integral vectors b. (5 Points)

Submission deadline: Tuesday, 21.1.2014, before the lecture.