Winter semester 2013/14 — Research Institute for Discrete Mathematics
Prof. Dr. S. Held — University of Bonn

Linear and Integer Optimization

Exercise Sheet 8

Exercise 8.1: Let

$$A := \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ s & -1 \end{pmatrix} \text{ and } b := \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}.$$

Use the Ellipsoid method to find for $x_0 = 0$, $R = 2$ and sufficiently large N a feasible solution in $P = \{x \in \mathbb{R}^2 : Ax \leq b\}$ for $s = -1$ and $s = -2$.

(5 Points)

Exercise 8.2: Show that the algorithm given in Theorem 7.20 solves the problem Linear Programming for an instance $\max\{c^\intercal x : Ax \leq b\}$, with $A \in \mathbb{Q}^{m\times n}, b \in \mathbb{Q}^m$, and $c \in \mathbb{Q}^n$, in $O((n+m)^9(\text{size}(A) + \text{size}(b) + \text{size}(c))^2)$ time. (5 Points)

Exercise 8.3: Let $E(A, x) \subset \mathbb{R}^n$ be an ellipsoid, $a \in \mathbb{R}^n$ and $E' = \{z \in E(A, x) : a^\intercal z \geq a^\intercal x\}$. Show that the ellipsoid $E(A', x')$ with $A' = \frac{n^2}{n^2-1}\left(A - \frac{2}{n+1} bb^\intercal\right)$, $x' = x + \frac{1}{n+1} b$ and $b = \frac{1}{\sqrt{a^\intercal A a}} Aa$ contains the half-ellipsoid E'.
Hint: You may prove and then use the Sherman-Morrison-Woodbury formula:
$(A - uv^\intercal)^{-1} = A^{-1} + \frac{A^{-1}uv^\intercal A^{-1}}{1 - v^\intercal A^{-1} u}$ if $v^\intercal A^{-1} u \neq 1$. (5 Points)

Exercise 8.4:
Let G be a simple graph. Show that the following problem can be solved in time polynomial in $|V(G)|$.

$$\begin{array}{llcll}
\min & \sum_{e=\{v,w\}\in E(G)} x_{vw} & & & \\
s.d. & \sum_{w\in S} x_{vw} & \geq & \left\lceil \frac{1}{4}|S|^2 + \frac{1}{2}|S| \right\rceil & (v \in V(G), S \subseteq V(G) \setminus \{v\}) \\
& x_{uw} & \leq & x_{uv} + x_{vw} & (u, v, w \in V(G)) \\
& x_{vw} & \geq & 0 & (v \in V(G)) \\
& x_{vv} & = & 0 & (v \in V(G))
\end{array}$$

(This is an LP-relaxation of the Optimal Linear Arrangement Problem: Find an ordering $\{v_1, \dots, v_{|V(G)|}\} = V(G)$ of the vertices such that $\sum_{\{v_i,v_j\}\in E(G)} |i - j|$ is minimum.) (5 Points)

Submission deadline: Tuesday, 10.12.2013, before the lecture.