Winter semester 2013/14
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University of Bonn

Linear and Integer Optimization

Exercise Sheet 3

Exercise 3.1: Let $A_1, \ldots, A_m \subseteq \mathbb{R}^n$ be convex sets. Show that if any $n+1$ of the sets have a non-empty intersection, then

$$\bigcap_{i=1}^{m} A_i \neq \emptyset.$$

Can you omit one of the requirements: **convexity**, any $n+1$ sets have a non-empty intersection, or the **finiteness** of the family $A_1, \ldots, A_m$? (4 Points)

Exercise 3.2: Prove the following transposition theorem:

$$\begin{gathered}(\exists x : Ax \leq c,\ Ax \neq c) \\ \dot{\vee} \\ \Big(\exists y : (A^T y = 0,\ c^T y = -1,\ y \geq 0) \vee (A^T y = 0,\ c^T y \leq 0,\ y > 0)\Big).\end{gathered}$$

(4 Points)

Exercise 3.3:

a) Prove the generalized Farkas Lemma (Lemma 4.1) from the lecture. (2 Points)

b) Let (P) be a linear program of the form $\min\{c^\intercal x : Ax \leq b\}$. Show that the dual of the dual is equivalent to (P). (3 Points)

Exercise 3.4: Consider the following linear program $\min\{c^\intercal x : Ax = b\}$. Show that it either does not have a solution, it is unbounded, or all feasible solutions are optimal. Does this statement hold if we additionally require $x \geq 0$? (3 Points)

Submission deadline: Tuesday, 6.11.2013, before the lecture.

Programming Exercise on the back!

Programming Exercise 1
Implement the Fourier-Motzkin Elimination to decide if an LP $\max\{c^\intercal x : Ax \leq b\}$ has a feasible solution. If it has a solution print a solution vector to the standard output as a single line. If it does not have a solution, print the string "`empty`" followed by a certificate vector according to Corollary 3.3 (in one line).
The program has to be implemented in C/C++ using the GNU compilers `gcc` or `g++`. The program should be run from the command line and read in a text file, whose name is given as an argument. The text file specifies the LP in the following format.

- The first line contains the number m of rows and n of columns of A.
- The second line contains n floating point numbers specifying c.
- The third line contains m floating point numbers specifying b.
- The next m lines contain the rows of A. Each line contains the n floating point numbers in the respective row.

On the web site to the exercises you find test instances and an example program in C for reading the input. You may use the example as a base for your implementation.
(10 Points)

Submission of the programming exercise until Tuesday, 20.11.2013, before the lecture via e-mail to your tutor **and** to held@or.uni-bonn.de!